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A STUDY ON DYNAMICS ANALYSIS OF 3-DOF R-R-R TYPE MANIPULATOR ARM

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Abstract- The aspect of this paper entitled "Dynamics of 3-DOF R-R-R Type Manipulator Arm" is to give the brief idea about the Dynamics is important for mechanical design, control, and simulation of the Manipulator Arm. Initially study has been done on the industrial robots, their applications, and the problems associated with the current robotic arms, and the work has been carried out on the solution of the problem. Literature Study has given ideas in the field of Mechanical design and Dynamics of linkages. 3-DOF non planer R-R-R type of manipulator is chosen for Mechanical design and Dynamics analysis. For structural design The dynamic equations of motion provide the relationships between actuation and contact forces acting on robot mechanisms, and the acceleration and motion trajectories that result.

Key words- robot, link mechanism, light weight, low cost, Deflection

"I. INTRODUCTION"

One of the primary objectives of robotics engineering is to design a manipulator capable of high link accelerations without sacrificing positional accuracy. Concern of current robotic research has been to develop anthropomorphic (human-like) arms capable of emulating the dexterity, manipulability, and workspace volume and payload-to-weight ratio of a human arm. The advent of composite materials, with very high stiffness-to-weight and strength-to-weight ratios as well as excellent damping properties, have made it possible for robotics engineers to build manipulators with excellent stiffness, strength, damping and low inertia.

1.1 Dynamics

Dynamics is important for mechanical design, control, and simulation. Inverse dynamics used in feed-forward control (required torques and forces computed).forward dynamics used for simulation. it determined joint acceleration based on torques and forces are specified .the joint-space inertia matrix is used in analysis in feedback control to line arise the dynamics and integral part of many forward dynamics formulation .and the operational-space inertia matrix used in control at the task at end effectors.

Dynamics provides the relationships between actuation and contact forces, and the acceleration and motion trajectories that result. The dynamic equations of motion provide the basis for a number of computational algorithms that are useful in mechanical design, control, and simulation.

1.1.1 Problems Associated with Current Robotic Arms

Most current robotic arms possess poor payload-to-weight ratios, poor damping and lack anthropomorphic manipulability and dexterity. Conventionally, to design a fast-moving arm required that the links have low inertia. Inevitably, this resulted in large end-effectors vibrations and long settling times. Conversely, to achieve high Positional accuracy required bulky, massive links. Due to the large inertia of the links, these robotic arms cannot move rapidly and require inordinate amounts of power. However, robot researchers the world over have already begun to offer many design solutions to these problems. To achieve the manipulability and dexterity of a human arm, innovative new joint mechanisms have been studied.

1.2 Design of the Manipulator Arm:

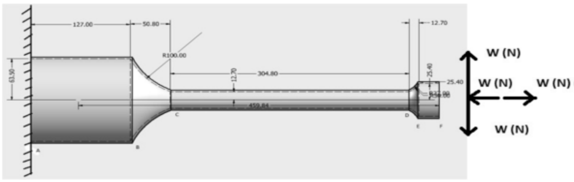

Figure1: Manipulator Arm Structure (Front View)

Consider a manipulator arm is the cantilever type of structure with payload to weight ratio 1:1with maximum deflection 1 CM. In fig.1 the structure is fixed at point A which will be attached to the robot base structure. Section AB is the hollow circular section of 3 mm thickness, which will be used to join the robot arm structure to the robot base structure. Section CD is the main arm structure which is hollow circular in cross section with 3 mm of thickness. Section EF is the hollow circular section of 3 mm thickness which will be used to fix the motor and the gripper mechanism. Further gripper will be attached to the section EF.

1.2.1 Generalized Forces:

The generalized force Q_1 is defined as : $Q_i = \sum_{j=1}^{N} F_j \left(\frac{\partial r_j}{\partial \Theta_i} \right)$ where Fj is the force at point j and rj is the position vector of point j. The index i correspond to Generalized coordinates.

1.2.2 Equations of Motions:

The equations of motion of a robot mechanism are usually presented in one of two Canonical forms:
the joint-space formulation, $I(\Theta)\Theta^{\bullet\bullet} + C(\Theta,\Theta^{\bullet})\Theta^{\bullet} + \tau_g(\Theta) = \tau$ or
the operational-space formulation, $\mathrm{A}(x)v^{\bullet} + \mu(x,v) + \rho(x) = f$

where f= the net force acting on a rigid body and is given by f = Ia + v$\times Iv$ where α = acceleration, v = velocity, These equations show the functional dependencies explicitly: I is a function of θ, **Λ** is a function of **x**, and so on. **x** is a vector of operational-space coordinates, while **v** and **f** are spatial vectors denoting the velocity of the end-effectors and the external force acting on it. If the robot is redundant, then the coefficients of this equation must be defined as functions of θ and $\Theta^{\bullet}$ rather than **x** and **v**.

1.3 Lagrange Formulation:

The two methods that are most commonly used in robotics are the Newton–Euler formulation and the Lagrange formulation. The former works directly with Newton's and Euler's equations for a rigid body, which are contained within the spatial equation of motion,. This formulation is especially enabled to the development of efficient recursive algorithms for dynamics computations.

The Lagrange formulation proceeds via the Lagrangian of the robot mechanism,

L = K.E – P.E

where K.E and P.E are the total kinetic and potential energy, respectively, of the mechanism.

The kinetic energy is given by $K.E = \frac{1}{2}\left[\Theta^{\bullet}\right]^{T}[I(\Theta)][\Theta^{\bullet}]$ The dynamic equations of motion can then be developed using Lagrange's equation for each generalized coordinate: Torque $\tau_i = \frac{d}{dt}\left(\frac{\partial(L)}{\partial \Theta^{\bullet}_i}\right) - \frac{\partial(L)}{\partial \Theta_i}$

The resulting equation can be written in scalar form: $\sum_{j=1}^{n} I_{ij}\Theta_j^{\bullet\bullet} + \sum_{j=1}^{n}\sum_{k=1}^{n} C_{ijk}\Theta_j^{\bullet}\Theta_k^{\bullet} + \tau_{gi} = \tau_i$

1.3.1 Lagrange-Euler Dynamic Model Algorithm for the Closed-Form Equation of Motion:

This algorithm carries out the complete dynamic formulation of an n-DOF manipulator that satisfies the condition for existence of closed-form geometric solutions. The various steps are

Step 1 Assign frames {0},....,{n}using DH notation such that frame {i} is oriented (aligned) with principle axis of link i.

Step 2 Obtain the link transformation matrix T_i^{i-1} for each link and from these compute product matrices T_2^0, T_3^0 and so on, which are required for computing the coefficients dij and its derivatives, using equation

Step 3 Define partial derivative velocity matrix Qi for each link, depending on weather the joint is revolute or prismatic.

Step 4 For each link i determine the inertia tensor Ii with respect to frame {i}

Step 5 Compute dij for i, j = 1, 2, ...n using equation $d_{ij} = \begin{cases} T_{j-1}^{0} Q_j T_i^{j-1} & for\ j \le i \\ 0 & for\ j > i \end{cases}$

Step 6 Compute the inertia coefficients Mij for i, j = 1, 2,.n using equation $M_{ij} = \sum_{p=\max(i,j)}^{n} Tr[d_{pj} I_p d_{pi}^T]$

Step 7 Compute the velocity coupling coefficients hijk for i, j, k = 1, 2, ..., n using equation

$$h_{ijk} = \sum_{p=\max(i,j,k)}^{n} Tr\left[\frac{\partial(d_{pk})}{\partial_{qp}} I_p d_{pi}^T\right] \text{And } \frac{\partial d_{ij}}{\partial_{qk}} = \begin{cases} T_{j-1}^{0} Q_j T_{k-1}^{j-1} Q_k T_i^{k-1} & for\ i \ge k \ge j \\ T_{k-1}^{0} Q_k T_{j-1}^{k-1} Q_j T_i^{j-1} & for\ i \ge j \ge k \\ 0 & for\ i < j\ or\ i < k \end{cases}$$

Step 8 Compute gravity loading terms Gi for each link, i = 1, 2, ..., n using equation

$$G_I = -\sum_{p=i}^{n} m_p g d_{pi}\ \bar{r}_p^{p}$$

Step 9 To formulate the i[th] equation for torque τu **,** substitute all the coefficients in equation

$$\tau_i = \sum_{j=1}^{n} M_{ij}(q)\ddot{q}_j + \sum h_{ijk} \dot{q}_j \dot{q}_k + G_i \quad for\ i = 1, 2, ...n$$

1.3.2 Dynamic Analysis Formulation for 3–DOF 4 Link RRR Type Manipulator Arm using Lagrange – Euler Approach:

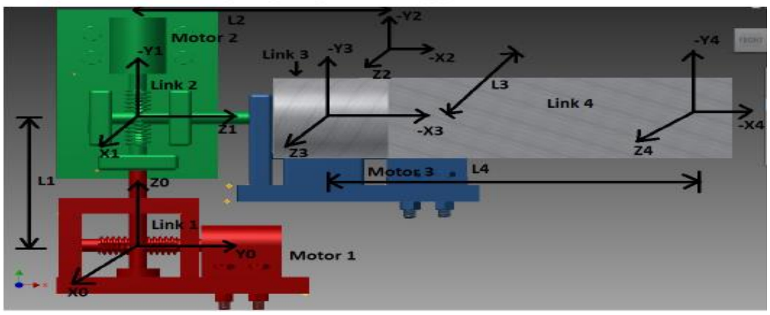

Fig1.1 Frame assignment for 3-DOF 4-Link RRR Type Manipulator Arm

The manipulator is show in the Fig. based on assumptions that all the four links, Link 1,Link 2, link 3, and Link 4 are cylindrical with mass m_1, m2 , m3, m4 respectively at their distal end, and Link 4 is rigidly connected with link 3.The Lagrange-Euler formulation is carried out to obtain the EOM as per Algorithm.The frames assignment is shown in Fig. 1 and Table 1 gives the joint link parameters.

i	Θ_i	d_i	a_i	α_i	$C\Theta_i$	$S\Theta_i$	$C\alpha_i$	$S\alpha_i$
1	Θ_1	L_1	0	-90	C_1	S_1	0	-1
2	Θ_2-90	L_2	0	-90	S_2	$-C_2$	0	-1
3	Θ_3	L_3	0	0	C_3	S_3	1	0
4	0	0	L_4	0	1	0	1	0

Table 1 Joint link parameters for 3-DOF 4-Link RRR Type Manipulator Arm

The link Transformation matrices and the overall transformation matrix are:

$T^0{}_1 = A_1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad T^1{}_2 = A_2 = \begin{bmatrix} S_2 & 0 & C_2 & 0 \\ -C_2 & 0 & S_2 & 0 \\ 0 & -1 & 0 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$T^2{}_3 = A_3 = \begin{bmatrix} C_2 & 0-S_2 & 0 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad T^3{}_4 = A_4 = \begin{bmatrix} 1 & 0 & 0 & -L_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$T^0{}_2 = T_1^0 T_2^1$, $T^0{}_3 = T_1^0 T_2^1 T_3^2$, $T^0{}_4 = T_1^0 T_2^1 T_3^2 T_4^2$

$T_3^1 = T_2^1 T_3^2$, $T_4^1 = T_2^1 T_3^2 T_4^3$, $T_4^2 = T_3^2 T_4^3$

Since all the four joints 1,2,3 and 4 are revolute joints. the velocity matrix is

Q1=Q2=Q3=Q4= $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ The coefficient d_{ij} for I, j =1,2,3,4

$d_{ij} = \begin{cases} T_{j-1}^0 Q_j T_u^{j-1} & for\ j \le i \\ 0 & for\ j > i \end{cases}$ which gives d_{11}, d_{12}, d_{13}, d_{14}, d_{21}, d_{22}, d_{23}, d_{24}, d_{31}, d_{32}, d_{33}, d_{34}, d_{41}, d_{42}, d_{43} and d_{44} as: $d_{11} = T_0^0 Q_1 T_1^0$, $d_{12} = d_{13} = d_{14} = d_{23} = d_{24} = d_{34} = 0$ (because j > i)

$d_{21} = T_0^0 Q_1 T_2^0$, $d_{22} = T_1^0 Q_2 T_2^1$, $d_{31} = T_0^0 Q_1 T_3^0 = 0$, $d_{32} = T_1^0 Q_2 T_3^1$

$d_{33} = T_2^0 Q_3 T_3^2$, $d_{41} = T_0^0 Q_1 T_4^0$,$d_{42} = T_1^0 Q_2 T_4^1$,$d_{43} = T_2^0 Q_3 T_4^2$ $d_{44} = T_3^0 Q_4 T_4^3$

The elements of inertia matrix M are computed

$$M_{ij} = \sum_{p=\max(i,j)}^{n} Tr\left[d_{pj} I_p d_{pi}^T\right] \quad \text{for } i,j = 1,2,3,4$$

$M_{11} = m_2 L_2^2 + m_2 (L_2^2 + L_3^2 C_2^2) + m_4 [(L_4 S_2 C_3 - L_3 C_2)^2 + (L_4 S_3 + L_2) \ ]$

$M \ \ = M \ \ = \qquad + \ [m_4 (L \ \ + L_4 S_3) (L_4 C_2 C_3 + L_3 S_2)]$

$M_{13} = M_{31} = m_4 [L_3 L_4 C_2 C_3 - L_2 \ L_4 \ S_2 S_3 - L_4{}^2 S_2]$

$M_{14} = M_{41} = m_4 [L_3 L_4 C_2 C_3 - L_2 \ L_4 s_2 s_3 - L_4{}^2 S_2]$

$M_{22} = m_3 L_3{}^2 + m_4 (L_3{}^2 + L_4^2 C_3^2)$ $M_{23} = M_{32} = - m_4 L_3 \ L_4 S_3$,

$M_{24} = M_{42} = - m_4 L_3 \ L_4 S_3$, $\quad M_{33} = m_4 L_4{}^2$, $M_{34} = M_{43} = m_4 L_4{}^2$, $M_{44} = m_4 L_4{}^2$

Thus, the Acceleration-Related Symmetric matrix M(Θ),will be

,M(Θ)= $\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}$

Substitute the values of the elements M_{11}, M_{12}, M_{13} to derive the Acceleration-Related Symmetric matrix M(Θ). The Coriolis and centrifugal force
coefficient h_{ijk} for I, j, k =1, 2, 3, 4 are computed

$$h_{ijk} = \sum_{p=\max(i,j,k)}^{n} Tr\left[\frac{\partial(d_{pk})}{\partial q_p} I_p d_{pi}^T\right]$$

$$h_{111} = Tr(T_0^0 Q_0 T_0^0 Q_1 T_1^0 I_1 d_{11}^T) = 0$$
$$h_{112} = h_{121} = Tr(T_0^0 Q_1 T_1^0 Q_2 T_2^1 I_2 d_{21}^T) = 0$$
$$h_{113} = h_{131} = Tr(T_0^0 Q_1 T_2^0 Q_3 T_3^2 I_3 d_{31}^T) = 0$$
$$h_{114} = h_{141} = Tr(T_0^0 Q_1 T_3^0 Q_4 T_4^3 I_4 d_{41}^T) = m_4 L_4 \left[L_2 C_3 + L_3 C_2 S_2 S_3 + L_4 C_2^2 S_3 C_3\right]$$

$$h_{122} = Tr(T_1^0 Q_2 T_1^1 Q_2 T_2^1 I_2 d_{21}^T) = 0$$,

$$h_{123} = h_{132} = Tr(T_1^0 Q_2 T_2^1 Q_3 T_3^2 I_3 d_{31}^T) = 0$$
$$h_{124} = h_{142} = -m_4 L_4 \left[C_2 S_3 (L_2 + L_4 S_3) + L_3 S_1 C_1 C_2^2 (C_3 + S_3)\right]$$
$$h_{133} = Tr(T_2^0 Q_3 T_2^2 Q_3 T_3^2 I_3 d_{31}^T) = 0$$
$$h_{134} = h_{143} = Tr(T_2^0 Q_3 T_3^2 Q_4 T_4^3 I_4 d_{41}^T) = m_4 L_4 S_2 C_3 \left[L_2 + L_3\right]$$
$$h_{144} = Tr(T_3^0 Q_4 T_3^2 Q_4 T_4^3 I_4 d_{41}^T) = -m_4 L_4 S_2 C_3 \left[L_2 + L_3\right] \quad h_{211} = Tr(T_0^0 Q_1 T_1^0 Q_2 T_2^1 I_2 d_{22}^T) = 0$$

$$h_{212} = h_{221} = Tr(T_0^0 Q_1 T_1^0 Q_2 T_2^1 I_2 d_{21}^T) = 0$$,

$h_{213} = h_{231} = 0$ $\quad h_{444} = 0$ $\quad h_{434} = h_{443} = 0$ $\quad h_{411} = 0$ $\quad h_{424} = h_{442} = 0$ $\quad h_{323} = h_{332} = 0$ $\quad h_{214} = h_{241} = 0$

$h_{223} = h_{232} = 0$ $\quad h_{412} = h_{421} = 0$ $\quad h_{344} = 0$ $\quad h_{324} = h_{342} = 0$

$h_{233} = 0$ $\quad h_{222} = 0$ $\quad h_{333} = 0$ $\quad h_{422} = 0$ $\quad h_{311} = 0$ $\quad h_{312} = h_{321} = 0$ $\quad h_{423} = h_{432} = 0$

$h_{313} = h_{331} = 0$ $\quad h_{334} = h_{343} = 0$ $\quad h_{314} = h_{341} = 0$ $\quad h_{322} = 0$ $\quad h_{433} = 0$

$h_{414} = h_{441} = 0$ $\quad h_{413} = h_{431} = 0$

$$Tr(T_0^0 Q_1 T_3^0 Q_4 T_4^3 I_4 d_{42}^T) = m_4 L_4 C_3 \left[L_3 S_2 + L_4 C_2 C_3\right]$$,

$$h_{224} = h_{242} = Tr(T_1^0 Q_2 T_3^1 Q_4 T_4^3 I_4 d_{42}^T) = h_{234} = h_{243} = Tr(T_2^0 Q_3 T_3^2 Q_4 T_4^3 I_4 d_{42}^T) = -m_4 L_3 L_4 C_3$$
$$h_{244} = Tr(T_3^0 Q_4 T_4^3 Q_4 T_4^3 I_4 d_{42}^T) = -m_4 L_3 L_4 C_3$$

The Coriolis and centrifugal torque terms are computed using the series summation:

$H_i = \sum\sum h_{ijk} \dot{q}_j \dot{q}_k$ for i,j,k = 1,2,3,4

Substituting the values of I, j, k in the above equation and simplifying gives

$H_1 = 2h_{114}\, \theta_1^\circ \theta_4^\circ + 2h_{124}\, \theta_2^\circ \theta_4^\circ + 2h_{134}\, \theta_3^\circ \theta_4^\circ + h_{114}\, \theta_4^{\circ 2}$

$H_2 = 2h_{214}\, \theta_1^\circ \theta_4^\circ + 2h_{224}\, \theta_2^\circ \theta_4^\circ + 2h_{234}\, \theta_3^\circ \theta_4^\circ + h_{244}\, \theta_4^{\circ 2}$ $\quad H_3 = 0 \quad H_4 = 0$

The mass of the links is at the distal end of the links {1} , {2}, {3}, {4}.Thus,

$r_1^{-1} = \begin{bmatrix}0 & 0 & L_1 & 1\end{bmatrix}^T$, $r_2^{-2} = \begin{bmatrix}0 & 0 & L_2 & 1\end{bmatrix}^T$, $r_3^{-3} = \begin{bmatrix}0 & 0 & L_3 & 1\end{bmatrix}^T$, $r_4^{-4} = \begin{bmatrix}0 & 0 & -L_4 & 1\end{bmatrix}^T$

And the gravity is in the –ve direction of Z-axis of frame {0}, that is , g = $\begin{bmatrix}0 & 0 & -g & 0\end{bmatrix}$

Where g = 9.0892 m/s^2 . Therefore the gravity loading G_1 at the joint 1 is ,

$G_1 = \left(m_1 g d_1 r_1^{-1} + m_2 g d_{21} r_2^{-2} + m_3 g d_3 r_3^{-3} + m_4 g d_4 r_4^{-4}\right)$, $\therefore G_1 = 0$

Similarly gravity loading at the joint 2,3, and 4 is G_2 ,G_3 and G_4 respectively,

$\therefore G_2 = \left(m_2 L_2 g C_2 + 2 m_3 L_3 g C_2 + m_4 (g C_4 (L_3 - L_4) - g L_4 S_2 C_3)\right)$ $\therefore G_3 = \left(m_4 L_4 g C_2 S_3\right)$

$$\therefore G_4 = \left(m_4 L_4 g C_2 S_3\right)$$

The gravity term matrix G_1 will be equal to

$$G_i = \left[G_1 \quad G_2 \quad G_3 \quad G_4\right]^T$$

Value of the term G_1, G_2, G_3 and G_4 can be substituted in the above equation to find the gravity matrix term,

$$\text{Thus,} G_i = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -[m_2 L_2 g C_2 + 2m_3 L_3 g C_2 + m_4 (g C_2 (L_3 - L_4) - g L_4 S_2 C_3)] \\ m_4 L_4 g C_2 S_3 \\ m_4 L_4 g C_2 S_3 \end{bmatrix}$$

Hence vector matrix form are obtained

$$[\tau_i] = [M(\theta)]\left[\ddot{\theta}_1\right] + H\left(\theta, \dot{\theta}\right) + (G_i)$$

$$[\tau_i] = \left[M_1\left(\ddot{\theta}_i\right)\right] + H_1\left(\dot{\theta}_i\right) + (G_i)$$

Thus substituting the values the equation of motion in the vector matrix form will be,

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} = [M(\theta)] \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{bmatrix}$$

$$\therefore \tau_1 = \left[m_2 L_2^2 + m_3\left(L_2^2 + L_3^2 C_2^2\right) + m_4\left[\left(L_4 S_2 C_3 - L_3 C_2\right)^2 + \left(L_4 S_3 + L_2\right)^2\right]\right]\ddot{\theta}_1 +$$
$$\left[m_3 S_2 L_2 L_3 + m_4\left(L_2 + L_4 S_3\right)\left(L C_2 C_3 + L_3 S_2\right)\right]\ddot{\theta}_2 + \left[m\left(L_3 L_4 C_2 C_3 - L_2 L_4 S_2 S_3 - L_4^2 S_2\right)\right]\ddot{\theta}_3 +$$
$$\left[m\left(L_3 L_4 C_2 C_3 - L_2 L_4 S_2 S_3 - L_4^2 S_2\right)\right]\ddot{\theta}_4 + 2h_{114}\dot{\theta}_1\dot{\theta}_4 + 2h_{124}\dot{\theta}_2\dot{\theta}_4 + 2h_{134}\dot{\theta}_3\dot{\theta}_4 + h_{144}\dot{\theta}_4^2$$

$$\therefore \tau_2 = \left[m_3 L_2 L_3 S_2 + m_4\left(L_2 + L_4 S_3\right)\left(L_4 C_2 C_3 + L_3 S_2\right]\ddot{\theta}_1 + \left[m_3 L_3^2 + m_4\left(L_3^2 + L_4^2 C_3^2\right)\right]\ddot{\theta}_2\right.$$
$$+ \left[m_4 L_3 L_4 S_3\right]\ddot{\theta}_3 - \left[m_4 L_3 L_4 S_3\right]\ddot{\theta}_4 + 2h_{214}\dot{\theta}_1\dot{\theta}_4 + 2h_{224}\dot{\theta}_2\dot{\theta}_4 + 2h_{234}\dot{\theta}_3\dot{\theta}_4 + h_{244}\dot{\theta}_4^2$$

$$\therefore \tau_3 = \left[m_4\left(L_3 L_4 C_2 C_3 - L_2 L_4 S_2 S_3 - L_4^2 S_2\right)\right]\ddot{\theta}_1 - \left[m_4 L_3 L_4 S_3\right]\ddot{\theta}_2 + \left[m_4 L_4^2\right]\ddot{\theta}_3 + \left[m_4 L_4^2\right]\ddot{\theta}_4 + m_4 L_4 g C_2 S_3$$
$$- \left[m_4 L_3 L_4 S_3\right]\ddot{\theta}_2 + \left[m_4 L_4^2\right]\ddot{\theta}_3 + \left[m_4 L_4^2\right]\ddot{\theta}_4 + m_4 L_4 g C_2 S_3$$

Due to the rigid joint between link 3 and 4, the value of the torque for link3, And link 4 are derived as equal in magnitude.
Power = Torque
Angular velocity = $\sum \tau\omega$
Power= $\tau_1\omega_1 + \tau_2\omega_2 + \tau_3\omega_3 = \tau_1\dot{\theta}_1 + \tau_2\dot{\theta}_2 + \tau_3\dot{\theta}_3$

1.4 Closure

Study of basic knowledge of dynamics was carried out. The necessary equations of motions were studied to gain the knowledge necessary for deriving the equations for dynamic analysis. Some of the important algorithms are summarized. The advantages of Lagrange approach is stated which motivated me to work with Lagrange Euler approach. This approach is used to formulate the torque equation for lifting a given load for a given angle.

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