Impact Factor (SJIF): 3.632

International Journal of Advance Research in Engineering, Science & Technology

e-ISSN: 2393-9877, p-ISSN: 2394-2444

Volume 3, Issue 6, June-2016

B-SPLINE COLLOCATION APPROACH TO A BOUNDARY LAYER FLOW WITH THERMAL RADIATION PAST A MOVING VERTICAL POROUS PLATE

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***Abstract* — *In this paper we examine the velocity and heat transfer in a boundary layer flow with thermal radiation past a moving vertical porous plate. The nonlinear unsteady momentum and energy equations associated with the flow and heat transfer are transformed from partial differential equations into ordinary differential equations by using similarity transformation. The resulting ordinary differential equation is solved numerically using B-Spine Collocation Method. The results are presented as velocity, temperature, Local wall shear stress and wall heat transfer rate profiles various values of parameter involving in the problem.*

Keywords*- Thermal radiation, porous plate, Velocity, Heat transfer, B-Spline Collocation method.*

I. INTRODUCTION

The study of the flow and heat transfer in fluid past a porous surface has attracted the interest of many scientific investigators in view of its applications in engineering practice, particularly in chemical industries; such as the cases of boundary layer control, transpiration cooling and gaseous diffusion.

Hayat *et al.* [5] investigated the hydro magnetic oscillatory flow of a fluid bounded by a porous plate when the entire system rotates about axis normal to the plate. And the result showed that the flow field is appreciably influenced by the material parameter of the second grade fluid, applied magnetic field, the imposed frequency, rotation, suction and blowing parameters**.**

Terrill [11] examined slow laminar flow in a converging or diverging channel with suction at one wall and blowing at the other wall. He considered the symmetric problem where the rate of fluid injection at one wall is equal the rate of fluid suction at the other wall.

Sivasankaran *et al.* [10] investigated the natural convection heat and mass transfer fluid past an inclined semi – infinite porous surface with heat generation using Lie group analysis. Their result revealed that the velocity and temperature of the fluid increases with the heat generation parameter. And also, the velocity of the fluid increases with the porosity parameter and temperature and concentration decreases with increase in the porosity parameter.

Ibrahim *et al.* [8] investigated the method of similarity reduction for problems of radiative and magnetic field effect on free convection and mass transfer flow past a semi-infinite flat plate. They obtained new similarity reductions and found an analytical solution for the uniform magnetic field by using lie group method. They also presented the numerical results for the non-uniform magnetic field.

Yurusoy and Pakdemirli [14] examine the exact solution of boundary layer equations of a non-Newtonian fluid over a stretching sheet by the method of lie group analysis and they found that the boundary layer thickness increases when the non-Newtonian behavior increases. They also compared the results with that of Newtonian fluid. Makinde [9] examined the free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. The plate is maintained at a uniform temperature with uniform species concentration and the fluid is considered to be gray, absorbing – emitting. The coupled non-linear momentum, energy and concentration equation governing the problem is obtained and made similar by introducing a time dependent length scale. The similarity equations are then solved numerically by using superposition method.

Chung [3] examined the nonlinear stability of steady flow and temperature distribution of a Newtonian fluid in a channel heated from below and the viscosity is a function of temperature.

Howell *et al.* [7] examined momentum and heat transfer on a continuously moving surface in a power law fluid .They examined the momentum and heat transfer occurring in the laminar boundary layer on a continuously moving and stretching two dimensional surface in non-Newtonian fluid. Their results in clued situation when then velocity is nonlinear and when the surface is stretched linearly.

Hassanien *et al*. [6] investigated the flow and heat transfer in a power law fluid over a non-isothermal stretching sheet. They presented a boundary layer analysis for the problem of flow and heat transfer from a power law fluid to a continuous stretching sheet with variable wall temperature. They performed parametric studies to investigate the effect of non-Newtonian flow index, generalized Prandtl number, power law surface temperature and surface mass transfer. Their result showed that friction factor and heat transfer depend strongly on the flow parameter.

Hassanien [6] examined the problem of a non-Newtonian viscoelastic fluid obeying the Walter's model with heat transfer over a continuous surface in parallel over a continuous surface in parallel stream using finite difference method.

Hassanien and Gorla [6] investigated the problem of the flow and heat transfer in a non-Newtonian fluid with micro – rotation past a stretching porous sheet.

Badr and Ahmed [1] investigated the numerical simulation of steady and unsteady mixed convection from tubes of elliptic cross-section. The problem of two-dimensional laminar mixed convection (forced and free convection) from a tube of elliptic cross-section was numerically simulated for the cases when the approaching flow is either steady or fluctuating. The elliptic cross-section is flexible enough to simulate the circular tube or to approach the flat plate depending on the ratio between its minor and major axes. The numerical scheme is developed in a way to cover the special cases of steady and unsteady flows, parallel and counter flows, forced convection as well as mixed convection flows. The tube is assumed to have an isothermal surface and is placed in an unsteady but uniform stream. The free - stream fluctuations are represented by periodic (sinusoidal) fluctuations superimposed on the average stream velocity. The resulting velocity and thermal fields are obtained by solving the conservation equations of mass, momentum and energy. The main parameters involved are the tube axis ratio, Reynolds number, Grashof number, Prandtl number, frequency parameter and the relative amplitude of fluctuations. Results will be presented for some steady flow cases covering the parallel, cross and counter flow configurations as well as the fluctuating flow cases with emphasis on the effects of the amplitude and frequency of free-stream fluctuations on the local and average Nusselt numbers. The study revealed that the effect of fluctuations on the time-average Nusselt number becomes more pronounced with increasing Reynolds number. It also revealed that the rate of heat transfer increases with the increase of the amplitude of fluctuations but decreases with the increase of frequency. The details of flow and thermal fields were presented in the form of local and average Nusselt number variations as well as streamline and temperature contours for some selected cases. This paper present the B-Spline Collocation solution of the velocity and heat transfer in a boundary layer flow with thermal radiation past a moving vertical porous plate. The method is based on the search for a solution in the form of a rapid convergent series.

II. MATHEMATICAL FORMULATIONS

Consider an unsteady flow of an incompressible fluid with thermal radiation past a moving vertical plate. Let the x- axis be taken along the plate in the vertically upwards direction and the y' – axis be taken normal to it. Let u and v be the velocity components along the x and y' the directions respectively. The physical variables are functions y' and t only. Hence, the appropriate governing equations are as follows:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y'} = \nu\frac{\partial^2 u}{\partial y'^2} + g\beta(T - T_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y'} = \alpha\frac{\partial^2 T}{\partial y'^2} + \frac{\alpha}{k}\frac{\partial q_r}{\partial y'} \tag{3}$$

and the radiative heat flux term is simplified by making use of the Roseland approximation as in equation (1)

$$q_r = -\frac{4\sigma}{3\delta}\frac{\partial T^4}{\partial y'} \tag{4}$$

where, α = thermal diffusivity, k = thermal conductivity, σ = Stefan – Boltzmann constant, K = Absorption Coefficient, ν = Kinematics viscosity, u,v = Velocity component, q_r = Radiative heat flux, β = Volumetric expansion coefficient = Temperature, G = Gravitational acceleration, t = Time, U_0 = Wall velocity, T_w = Wall Temperature, T_∞ = Ambient temperature, c = Suction parameter, R = Radiation parameter, G_r = Local Grashof number, P_r = Prandtl number.

The appropriate boundary conditions are:

$$\left.\begin{array}{lll} u=U_0, & T=T_w & at \quad y'=0 \\ u\to 0, & T=T_\infty & \text{as} \quad y'\to\infty \end{array}\right\} \tag{5}$$

III. METHOD OF SOLUTION

We define similarity variables

$$y=\frac{y'}{\sqrt{vt}}$$

where the length scale

$$\delta=2\sqrt{vt}, u=U_0 f(y), \theta(y)=\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

By Taylor series expansion of T^4 and neglecting terms with higher powers, we have

$$T^4\approx 4TT_\infty^3-3T_\infty^4 \tag{7}$$

Using equations (6) and (7) in equations (2) to (4) we obtain,

$$f''=-2(c+y)f'-G_r\theta \tag{8}$$

$$\theta''=-\frac{2P_r}{(1+R)}(c+y)\theta' \tag{9}$$

With

$$f(0)=\theta(0)=1 \qquad f(5)=\theta(5)=0 \tag{10}$$

Where

$$R=\frac{16\sigma\alpha T_\infty^3}{3kK} \qquad G_r=\frac{g\beta(T_w-T_\infty)}{vU_0} \qquad P_r=\frac{v}{\alpha} \tag{11}$$

Local wall shear stress, $\tau_w=\left[\mu\frac{\partial u}{\partial y}\right]_{y=0}$

i.e. skin friction = $\frac{\tau_w\delta}{\mu U_0}=f'(0)$ (12)

and

Local surface heat flux $q_w=\left[-k\frac{\partial T}{\partial y}\right]_{y=0}-\left[\frac{4\sigma}{3K}\frac{\partial T^4}{\partial y}\right]_{y=0}$ (13)

$$N_u=\frac{q_r\delta}{k(T_w-T_\infty)(1+R)}=-\theta'(0) \tag{14}$$

IV. B-SPLINE COLLOCATION METHOD

Let $\{t_i\}_{i=0}^n$ be a strictly increasing sequence of point such that $a=t_0<t_1<........<t_n$. Consider that f(t) is a continuous function over an interval [a,b]. A polynomial spline s(t) of degree m is an interpolant to f(t), such that

(i) s(t) is a polynomial of degree m over each subinterval $[t_i, t_{i+1}]$, i=0,1,2,…..,n-1.

(ii) $s(t_i)=f(t_i), \qquad i=0,1,2,......,n$

(iii) $s(t)\in\subset^{m-1}[a,b]$

Prenter (1975) defined the cubic spline s(t) as

$$s(t)=\sum_{j=-1}^{n+1}a_jB_j(t) \tag{15}$$

Where n is the number of subintervals of [a,b], $a_{-1}, a_0, ..., a_{n+1}$ are (n+3) unknowns and the functions $B_i(t)$, with additional knots $t_{-2} < t_{-1} < t_0$ and $t_{n+2} > t_{n+1} > t_n$ is defined by,

$$B_i(t) = \frac{1}{h^3}\begin{cases} (t-t_{i-2})^3; & if \;\; t \in [t_{i-2}, t_{i-1}] \\ h^3 + 3h^2(t-t_{i-1}) + 3h(t-t_{i-1})^2 - 3(t-t_{i-1})^3; if \;\; t \in [t_{i-1}, t_i] \\ h^3 + 3h^2(t_{i+1}-t) + 3h(t_{i+1}-t)^2 - 3(t_{i+1}-t)^3; if \;\; t \in [t_i, t_{i+1}] \\ (t_{i+2}-t)^3; if \;\; t \in [t_{i+1}, t_{i+2}] \\ 0; & otherwise \end{cases}$$

$$B_i(t_j) = \begin{cases} 4; & if \;\; j = 1 \\ 1; & if \;\; j = i-1 \;\; or \;\; i+1 \\ 0; & if \;\; j = i-2 \;\; or \;\; i+2 \end{cases}$$

And $B_i(t) = 0$ for $t \geq t_{i+2}$ and $t \leq t_{i-2}$.

Here $B_i(t)$ is denoted as cubic B-spline with knots at $t_{-2} < t_{-1} < .... < t_n < t_{n+1} < t_{n+2}$. From equation (15) it is clear that s (t) is a basis of B-splines at different knots.

Let us consider a general two point boundary value problem of second order is as given below, (16)

$$\theta''(t) + p(t)\theta'(t) + q(t)\theta(t) = r(t) \quad for \quad a \leq t \leq b$$

boundary conditions are,

$$A_0\theta(a) + D_0\theta'(a) = \alpha \tag{16a}$$

$$A_n\theta(b) + D_n\theta'(b) = \beta \tag{16b}$$

To solve this problem by B-Spline Collocation, Let $a = t_0 < t_1 < .... < t_n = b$ be a uniform partition of the interval [a,b] and $t_{i+1} - t_i = h = 1/n$; where n is the number of subinterval of [a,b].

We seek a function

$$\theta(t) = \sum_{j=-1}^{N+1} a_j B_j(t) \tag{17}$$

Approximating the solution $\theta(t)$ to the equation (9). Where $B_j(t)$ be cubic B-Spline defined as in Table-1 and $a_j, j = -1, 0, ..., n+1$ are the unknowns to be determined. Substituting $\theta(t)$ in the equation (9) and (10) we obtain

$$\sum_{j=-1}^{n+1} a_j [B_j''(t_i) + p_i B_j'(t_i) + q_i B_j(t_i)] = t_i \tag{18}$$

where $p_i = p(t_i), q_i = q(t_i) \qquad i = 0, 1, ...n$

$$\sum_{j=-1}^{n+1} a_j [a_0 B_j(t_0) + D_0 B_j'(t_0)] = \alpha \tag{18a}$$

$$\sum_{j=-1}^{n+1} a_j [a_n B_j(t_n) + D_n B_j'(t_n)] = \beta \tag{18b}$$

Evaluating $B_j(t)$ and its derivative at the knots of our partition using Table-1by Dr.D.C.Joshi [4], substituting into the collocation equations (17), (17a) and (17b). We arrive at the system of n+3 unknowns a_{-1} , a_0 , ..., a_{n+1} .

Now eliminating a_{-1} from the first two equations and Similarly eliminating a_{n+1} from the last two equations we obtain the system of (n+1) linear equations in (n+1) unknowns a_0 , a_1 , ..., a_n. we can solve the system of equations for a_0 , a_1 , ..., a_n and using it also find a_{-1} and a_{n+1}. Hence the method of collocation applied to equations (15), (15a) and (15b) using a basis of cubic B-Spline has a complete solution $\theta(t)$. So, the solution of equation (8) is also obtained by the similar process.

Table: 1 Values of cubic B-Spines at notes

	t_{j-2}	t_{j-1}	t_j	t_{j+1}	t_{j+2}
$B_j(t)$	**0**	**1**	**4**	**1**	**0**
$B_j^{'}(t)$	**0**	$3/h$	**0**	$-3/h$	**0**
$B_j^{''}(t)$	**0**	$6/h^2$	$-12/h^2$	$6/h^2$	**0**

The result is presented graphically in figures 1 to 6.

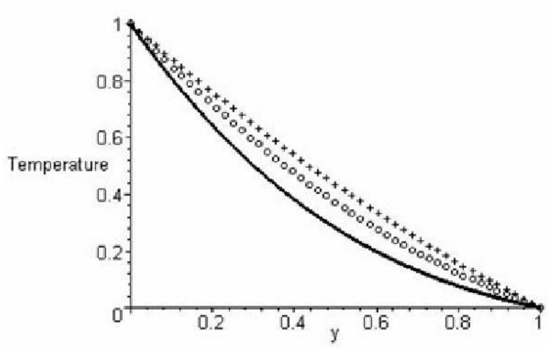

Figure 1: Temperature profiles, c=1, Gr=10, Pr=0.71, ______R=0.1, ooooooo R=1.0, +++++++ R=3.0

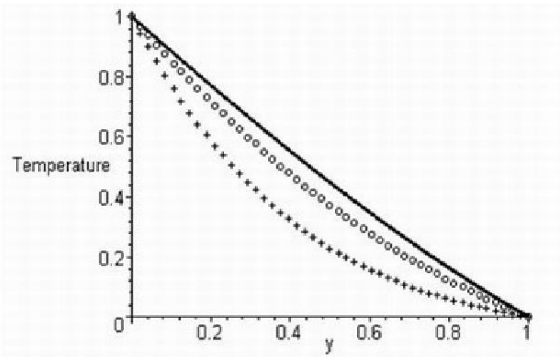

Figure 2: Temperature profiles, R=1, Gr=10, Pr=0.71, ______c=0.1, ooooooo c=1.0, +++++++ c=3.0

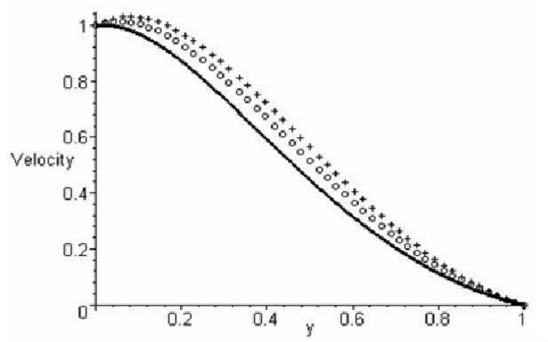

Figure 3: Velocity profiles, c=1, Gr=10, Pr=0.71, ______R=0.1, ooooooo R=1.0, +++++++ R=3.0

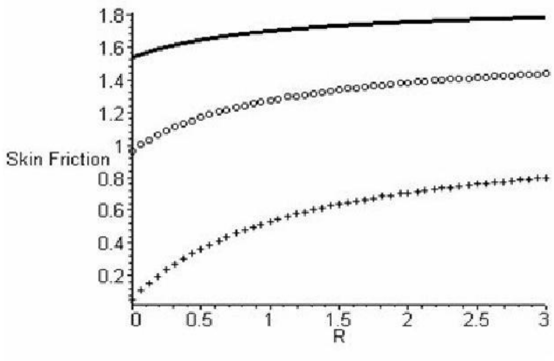

Figure 5: Wall shear stress, Gr=10 Pr=0.71, ______c=0.1, ooooooo c=0.5, +++++++ c=1.0

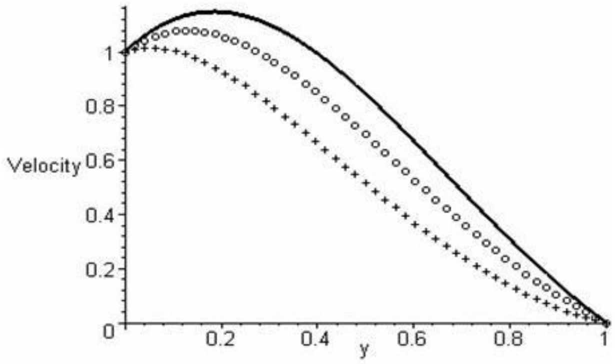

Figure 4: Velocity profiles, R=1, Gr=10, Pr=0.71, ______c=0.1, ooooooo c=0.5, +++++++ c=1.0

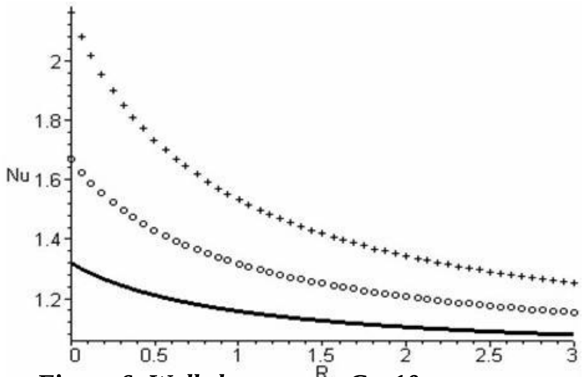

Figure 6: Wall shear stress, Gr=10, Pr=0.71, ______c=0.1, ooooooo c=0.5, +++++++c=1.0

V. DISCUSSIONS OF RESULTS

The result is presented as temperature, velocity, wall shear stress and wall heat transfer rate profiles in figures 1 to 6. And the results show that:

The fluid temperature is maximum at the plate and decreases as the fluid is moving away from the plate. Increase in the radiation heat absorption causes increases in the fluid temperature. And the fluid temperature decrease with an increase in fluid suction at the plate.

The fluid velocity reaches its maximum value at short distance from the plate and decreases to zero value away from the plate. An increase in the fluid velocity is observed with an increase in the radiative heat absorption. And the fluid velocity decreases with increase in the fluid suction at the plate.

Generally, the skin friction increases with decrease radiative heat absorption but decrease with increases in the fluid suction the plate. Thus, suction reduces skin friction at the plate. The rate of heat transfer at the plate decreases with an increase in the radiative heat absorption but decreases with an increase in fluid suction at the plate.

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