Impact Factor (SJIF): 3.632

International Journal of Advance Research in Engineering, Science & Technology

e-ISSN: 2393-9877, p-ISSN: 2394-2444
Volume 3, Issue 4, April-2016

Comparative Study of Curved Beam Using Finite Element Analysis (FEA) and Isogeometric Analysis (IGA)

Siddharth A. Moteriya[1], Vishal B. Patel[2], Vishal A. Arekar[2]

[1] *Research Scholar, Structural Engg, Dept., Birla Vishvakarma Mahavidhyalaya*
[2] *Assistant Professor, Structural Engg. Dept., Birla Vishvakarma Mahavidhyalaya*

Abstract: In the real world of designing field the different geometries are generated using the Computer Aided Design (CAD). And then after this geometries are analysed using different analytical methods like Finite Element Method (FEM), Finite Difference Method and other traditional methods. Among them Finite Element Method is more popular in recent time among the designers. But, FEM consumes more time in preparing the boundary of the element. Hence, it consumes more time to get final results. In last decade, a new emerging technology named as Isogeometric Analysis has been developed by Hughes in 2005. As compared to FEM, the Isogeometric Analysis prepares the meshing 80% faster than FEM. So, from the literature, this study is concentrated on the comparison of Isogeometric Analysis with FEM. In this research work, the main aim is to compare Isogeometric Analysis with Finite Element Analysis using Isogeometric tool. In this study, two models of Beam Curved in Plan have been analysed and results of Finite Element Analysis and Isogeometric Analysis using ABAQUS 6.14 software have been compared.

Keywords: Isogeometric Analysis (IGA), Finite Element Analysis (FEA), FEM, Curved Beam, B-Spline, Non-Uniform B-Splines (NURBS), Equivalent Stresses (Von Mises), Maximum and Minimum Principal Stress, ABAQUS.

I. INTRODUCTION

In real word of designing the engineering problems includes analysis on products like aircraft, automobiles, boats, wind turbines and components of these products. The geometry of all these products are described using Computer Aided Design (CAD).

In the field of engineering a lot of time is waste on approximating this geometry for analysis purposes. Most of the analysis methods are uses piecewise linear or piecewise quadratic approximations of the boundary. Since such an approximation is not unique, engineers waste time going back and forth between these two definitions of the geometry. Traditionally, geometry has been represented differentially in the field of CAD and FEA. This means that the CAD geometry, which is exact, must be converted to an Analysis Suitable Geometry (ASG) for input into a FEA programme. In order to obtain an ASG, features like insert holes and another difficult boundaries are often omitted to avoid for making analysis easy and simple during analysis process. This process takes up to 80% analysis time and this is also known as 'Meshing".

The difference between the fields of FEA and CAD on the subject of geometry representation is remarkable. This has mainly to do with the fact that they are seen as separate fields, which are interfaced using complicated and expensive mesh generation schemes. In order to avoid this problem, it is preferable to use an integrated approach where the CAD geometry is directly used in the FEA. Some attempts at this have been made in the recent years but among that the Isogeometric Analysis Method is very good and gives user interfaced results.

To overcome this critical issues in FEA, an idea has been developed by Hughes et al. in 2005 that, In the analysis framework, it is employed the same function used to describe the geometry of the computational domain i.e. typically use B-Splines and/or Non–Uniform Rational B-Splines (NURBS).

II. ISOGEOMETRIC ANALYSIS

Isogeometric analysis was introduced in 2005 by Hughes et al. to get exact engineering geometry to Finite Element Analysis (FEA) and to mitigate the inconvenient process of meshing altogether.

The key concept of IGA outlined by Hughes et al. in 2005 is, "To employ Non-Uniform Rational B-Splines (NURBS) not only as a geometry discretization technology but also a discretization tool for analysis". The IGA concept merge the two fields of CAD and FEA by expanding the solution space using the same basis at that of the geometry description from CAD. Since, its introduction, IGA has successfully applied to a wide variety of problems in structural analysis, electromagnetics, turbulence, fluid structure interaction and higher order partial differential equations.

There are several candidate technologies available to the IGA framework, of which NURBS is most commonly used tool since it is standard method employed in CAD programs. NURBS generalizes B-Splines and consequently inherit all of their favourable properties for free from design. NURBS are commonly used in Computer Aided Design (CAD), Computer Aided Manufacturing (CAM) and Computer Aided Engineering (CAE).

Isogeometric Analysis is based on NURBS, has refinement procedure related to h-refinement, p-refinement which are respectively known as Knot Intersection and Degree Elevation. Hence, Isogeometric Analysis has advantages like:

- Directly interacting with the CAD systems
- Greatly simplifying the refinement processes
- Improving the solution accuracy
- Reducing the computational costs and time

III. B-Splines & NURBS

A. B-Splines

Knot Vector: A knot vector in one dimension is a non-decreasing set of coordinates in the parameter space, written = $\{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$.

Where, $\xi i \in R$ is the i^{th} knot,
i= the knot index, i = 1, 2, . . . , n + p + 1,
p = the polynomial order, and
n = the number of basis functions used to construct the B-spline curve.

Knot vectors may be *uniform* if the knots are equally spaced in the parameter space.
Knot vector may be *non-uniform* if they are unequally spaced in the parameter space.
A knot vector is said to be *open* if its first and last knot values appear p + 1 times.

B. Parametric Domain of B-Splines

B-splines are defined on a parameter space Ω´. The B-Spline parameter space is local to "patches" instead of elements, where the patch can be seen as a "macro-element". The parameter domain itself is defined by the knot vector(s) Ξ. The knot vector is defined as under,

$\Xi = \{(\xi_1, \ldots, \xi_{p+1} = a),\ \xi_{p+2}, \ldots, \xi_n, (\xi_{n+1}, \ldots, \xi_{n+p+1} = b)\}$

Where, $\xi i \in R$ is the i^{th} knot,
i is the knot index, i = 1, 2, . . . , n + p + 1,
p is the polynomial order, and
n is the number of basis functions used to construct the B-spline curve.

Higher Dimensional parameter space are constructed using a tensor product of 1D knot vectors. Hence the parametric domains are defined by the set $[a, b]^d \in R^d$ with d is the dimension of the space. Using the knot vector we can construct B-Spline basis function of order p+1 which are piecewise polynomials of degree p. Repeated knots are allowed, hence $\xi_1 \le \xi \le \cdots \le \xi_{n+p+1}$. A knot which is repeated k times in knot vector is said to have a multiplicity k.

C. B-Spline Basis Function

The B-Spline basis function are defined recursively starting with piecewise constants (*p* = 0):

$$B_{i,p} = \begin{Bmatrix} 1 & if\ \xi_i \le \xi \le \xi_{i+1} \\ 0 & otherwise \end{Bmatrix} \quad (1)$$

For polynomial order p=1, 2, 3,… they are defined by,

$$B_{i,0}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} B_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} B_{i+1,p-1}(\xi) \quad (2)$$

So given a knot vector and a polynomial degree the B-Spline function space B is uniquely defined as

$$B \equiv B(\Xi; p) := span\ \{B_{i,p}\}_{i=1}^{n}$$

The result of equation (1) and (2) is shown in Figure 1 for the knot vector $\Xi = \{0,0,0,0,1,2,3,4,4,4,4\}$. Figure 1 shows Recursive generation of a cubic basis for the uniform knot vector $\Xi = \{0,0,0,0,1,2,3,4,4,4,4\}$. An example of a quadratic basis for an open, non-uniform knot vector is shown in Fig.-3.2. Here the implications of the repeated knots at the ends of the interval and also at $\xi = 4$ are shown, where the continuity is lowered to C^0. The other basis functions are C^1 continuous. Degree p basis functions have up to p−1 continuous derivatives. A repeated knot will reduce the number of continuous derivatives by 1. When the multiplicity equals p, the basis function is nodal. `

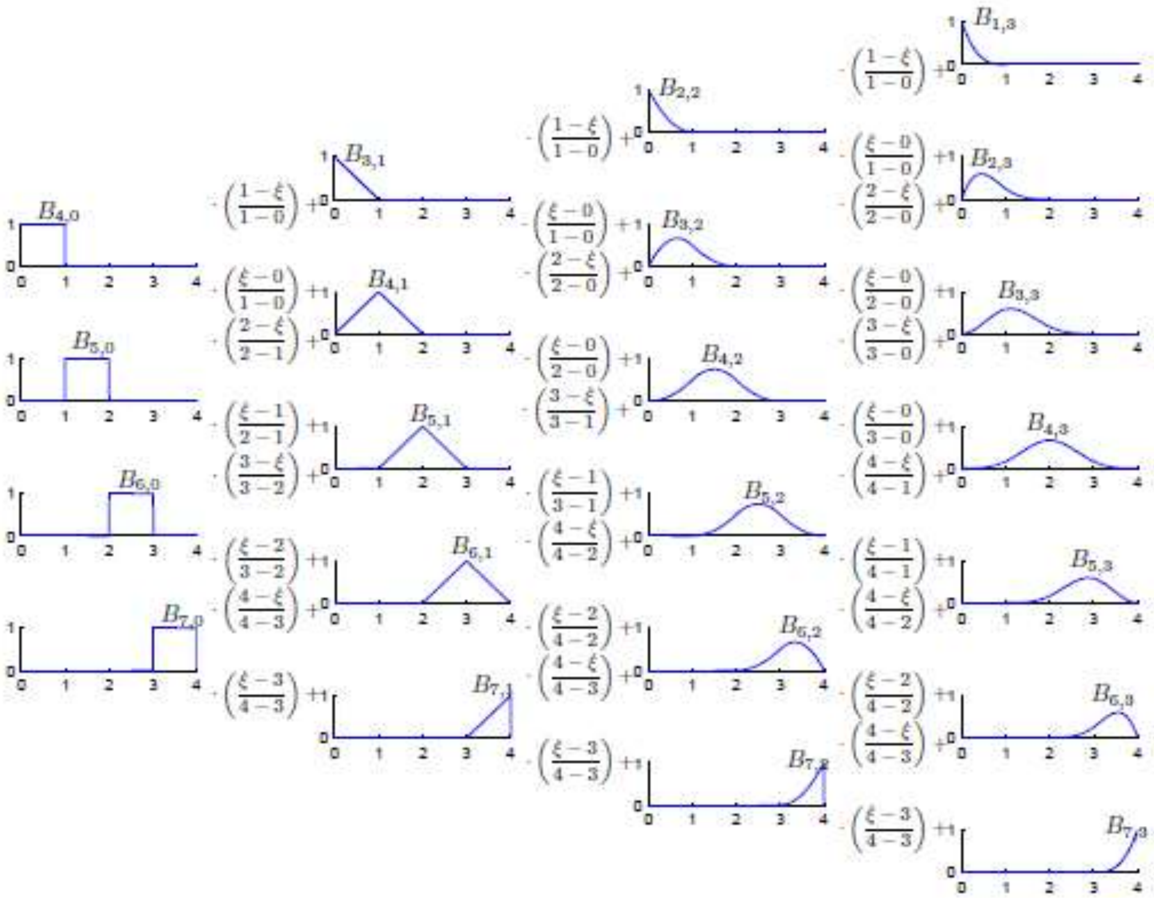


Figure 1. B-Spline Basis Function

The basis functions possess the following important properties:

a) Non-negativity: $B_{i,p}(\xi) \geq 0$ for $\forall i, p$ and $a \leq \xi \leq b$.

b) On a knot span $[\xi_i, \xi_{i+1})$ there are p+1 non-zero functions.

c) Partition of Unity. $\sum_{i=1}^{n} B_{i,p}(\xi) = 1$.

d) The basis function from a linear independent basis which makes them suitable for analysis.

e) $B_{0,p}(0) = B_{n,p}(1) = 1$.

f) For compact support$[\xi_i, \xi_{i+p+1})$. Higher order functions have support across larger portions of the domain. This increase in support has no implications on the bandwidth of the resulting linear system in numerical applications. The total number of functions that any function shares support with (including the function itself) is 2p+1 which is equal to that for Lagrange polynomials.

D. Non-Uniform Rational B-Splines (NURBS):

B-splines have their rational counterparts giving the ability to exactly represent objects that cannot be represented by polynomials. For example in CAD circular and conic shapes are often used, which can be exactly represented by NURBS.

E. NURBS basis functions

The NURBS basis is defined by associating the B-spline basis functions with a strictly positive weight, w_i as

$$N_{i,p}(\xi) = \frac{w_i B_{i,p}(\xi)}{W(\xi)} \text{ Where, } W(\xi) = \sum_{i=1}^{n} w_i B_{i,p}(\xi)$$

Spanning the NURBS functions space uniquely defined as N≡ $\mathcal{N}$ (Ξ; p; w) := span $\{N_{i,p}\}^{n}_{i=1}$. Analogous to B-Splines higher dimensional function spaces are constructed using tensor products of univariate basis functions N≡ $\mathcal{N}$ (Ξ, Ħ, …; p, q, …; w) := span$\{ N_{i,p} \otimes N_{j,q} \otimes \ldots \}_{i,j,\ldots=1}^{n,m,\ldots}$.

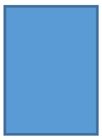
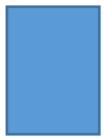
The NURBS basis has the following properties:

1. The NURBS basis constitutes a partition of unity $\sum_{i=1}^{n} N_{i,p}(\xi) = 1 \ \forall\xi$.
2. NURBS inherit their properties from the B-Spline basis functions like continuity across knots, local support and non-negativity.
3. The NURBS basis functions are not polynomial but rational functions.
4. If the weights are equal the basis is again polynomial. Hence, B-Splines are a special case of NURBS.

IV. MODEL INFORMATION

Isogeometric Analysis gets effective in curvilinear element where meshing is complex problem. This study presents the analysis of Beam Curved in Plan (2 Models – Quarter Circle and Semi Circular Models). The general model information are given in following table:

Table 1. Curve Beam Model Information

	Model-1	**Model-2**
Type	Quarter Circular Beam	Semi Circular Beam
Cross Section		
C/S Size	230 X 300 mm	230 X 300 mm
Centroidal Radius	4000 mm	4000 mm
Length	6283.185 mm	12566.370 mm
Applied Failure Pressure	0.901 N/mm^2	0.192 N/mm^2
Nodes	621	344
Elements	64	28
Element Types	20 Noded hexahedron	20 Noded hexahedron

V. FEA RESULT OF CURVED BEAM

The finite element analysis of Beam Curved in Plan (2 Models) are carried out and their different results like Equivalent Stress (Von Mises), Maximum Principal Stress, Minimum Principal Stress and Total Deformation are listed out using the FEM software tool Simulia Abaqus 6.14 version.

A. FEA Result of Curved Beam Model-1:

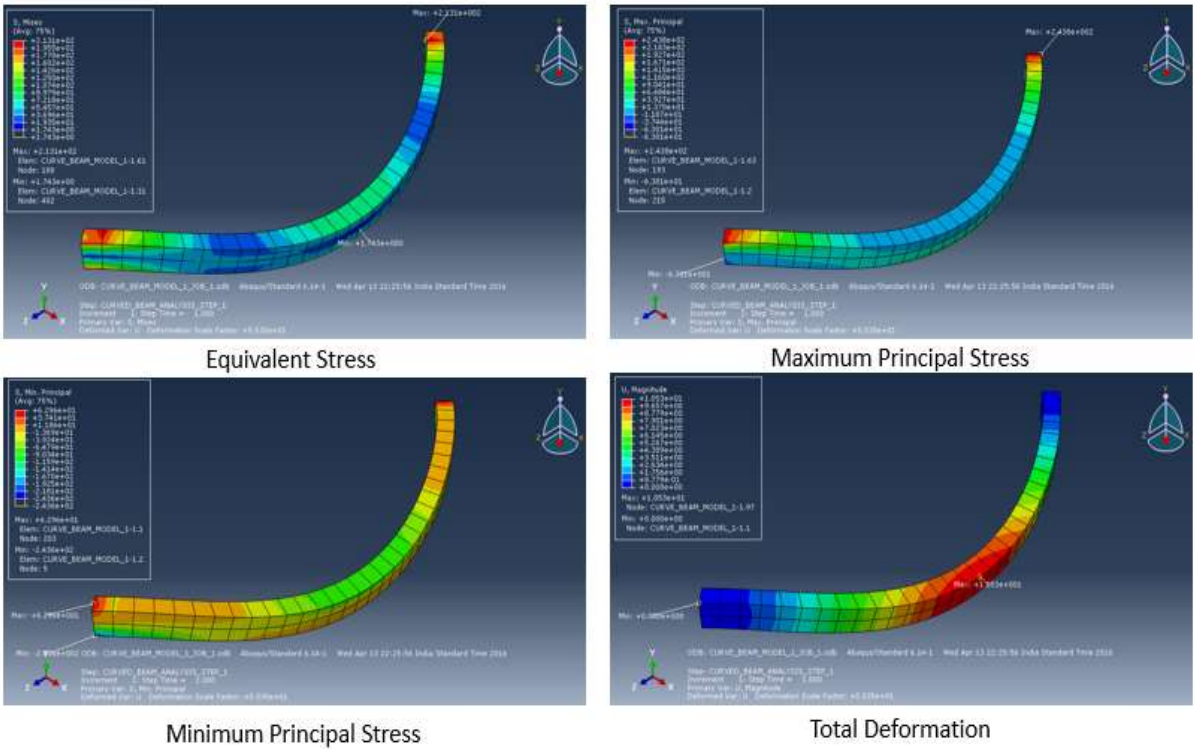


Figure 2. FEA Result of Model-1

B. FEA Result of Curved Beam Model-2:

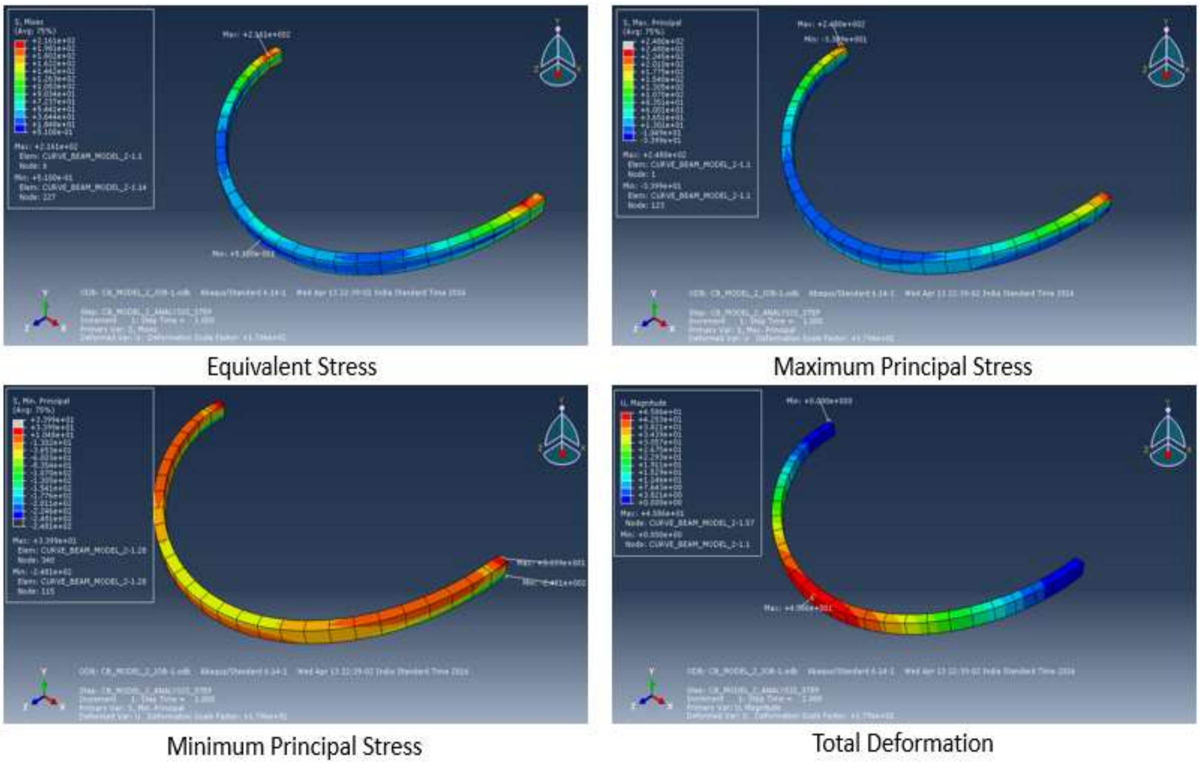


Figure 3. FEA Result of Model-2

C. FEA Result of Curved Beam:

Table 2. FEA Result Comparison of Model-1 & Model-2

	Model-1	Model-2
Equivalent Stress (N/mm^2)	213.1	216.1
Maximum Principal Stress (N/mm^2)	243.8	248.00
Minimum Principal Stress (N/mm^2)	243.6	248.1
Total Deformation (mm)	10.53	45.86

VI. IGA RESULTS OF CURVED BEAM

The finite element analysis of Beam Curved in Plan (2 Models) are carried out and their different results like Equivalent Stress (Von Mises), Maximum Principal Stress, Minimum Principal Stress and Total Deformation are listed out using the FEM software tool Simulia Abaqus 6.14 version using IGA tool NURBS Plugins.

A. IGA Result of Curved Beam Model-1:

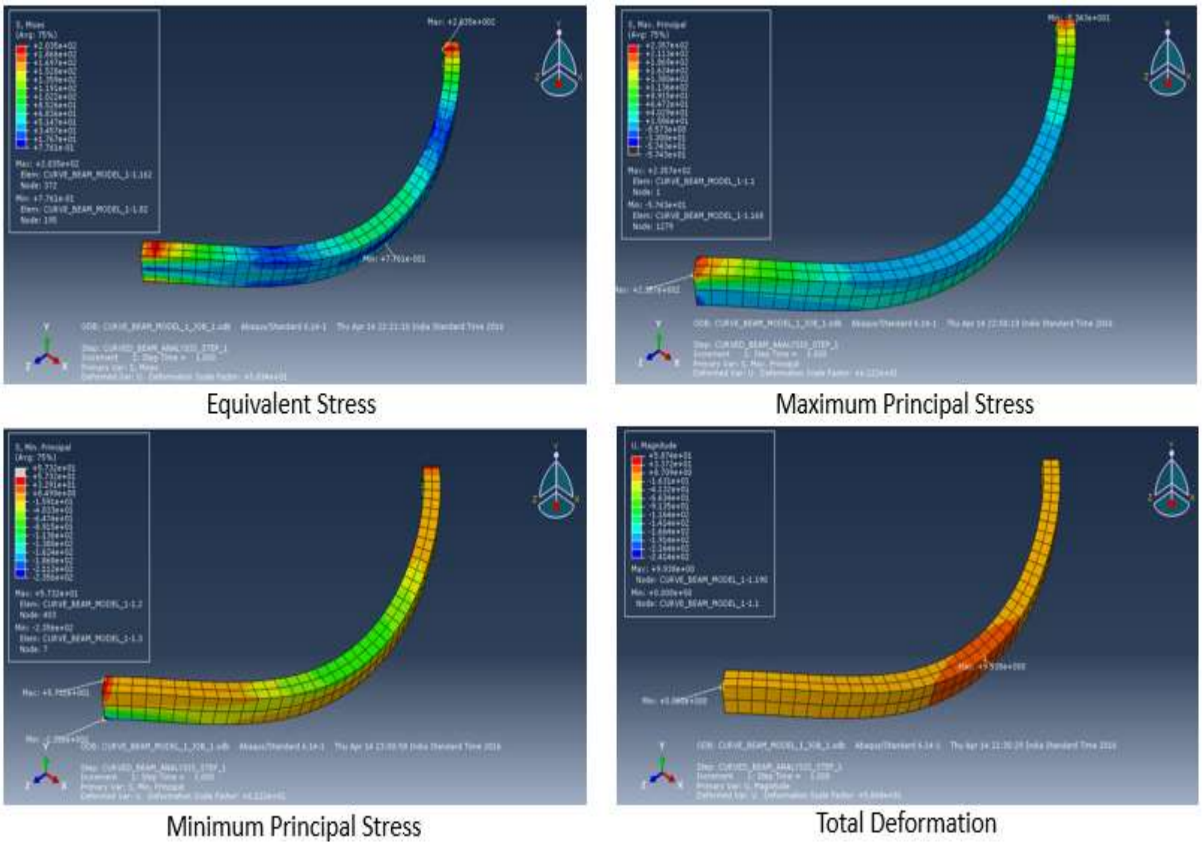


Figure 4. IGA Result of Model-1

B. IGA Result of Curved Beam Model-2:

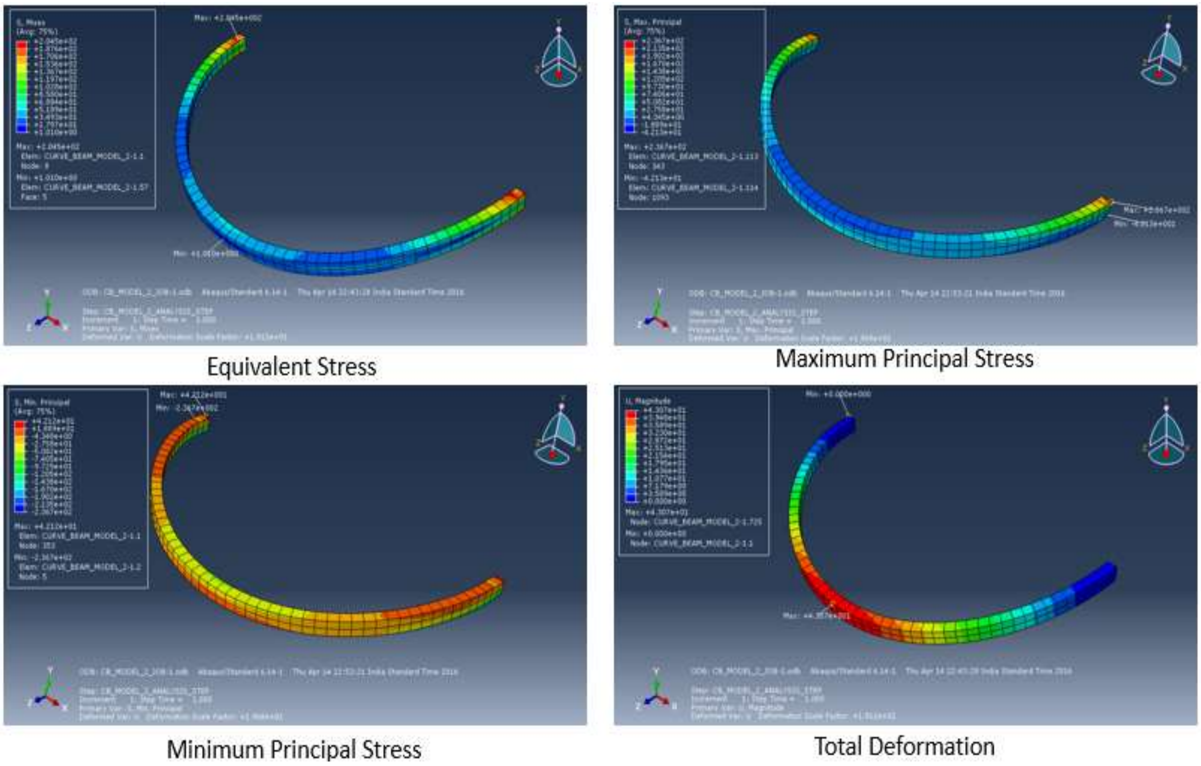


Figure 5. IGA Result of Model-2

C. IGA Result of Curved Beam:

Table 3. IGA Result Comparison of Model-1 & Model-2

	Model-1	**Model-2**
Equivalent Stress (N/mm^2)	203.5	204.5
Maximum Principal Stress (N/mm^2)	235.7	236.7
Minimum Principal Stress (N/mm^2)	235.6	236.7
Total Deformation (mm)	9.938	43.07

VII. COMPARISON OF FEA AND IGA FOR CURVED BEAM MODELS

The variation in FEA and IGA results are listed in following table:

Table 4. Percentage Difference in FEA & IGA Results

	Model-1	**Model-2**
Equivalent Stress (%)	4.50	5.36
Maximum Principal Stress (%)	3.32	4.556
Minimum Principal Stress (%)	3.28	4.594
Total Deformation (%)	5.62	6.08

VIII. CONCLUSION

The main focus of this study is to understand the behaviour of curvilinear elements in terms of stresses and deformation. It is known to us that the FEA gives a good satisfactory results for any curved or complex geometry. But during literature review it has been found that IGA may give more perfect and satisfactory results than FEA.
In this study, the behaviour of beam curved in plan has been studied and comparative result between FEA and IGA has been presented. From these result it has been concluded that the IGA gives less value for stress and deformation than the FEA rather keeping same loading condition.
Comparative results for beam curved in plan are as under:

- For Quarter Circle Curve Beam, IGA values for Equivalent Stress (Von-Mises), Maximum Principal Stress, Minimum Principal Stress and Total Deformation gives 4.50%, 3.32%, 3.28%, 5.62% respectively less than FEA values for same loads.
- For Semi Circular Curve Beam, IGA values for Equivalent Stress (Von-Mises), Maximum Principal Stress, Minimum Principal Stress and Total Deformation gives 5.36%, 4.50%, 4.59%, 6.08% respectively less than FEA values for same loads.

As shown in advantage of IGA, IGA improves the solution accuracy and directly integrate with CAD model. From this conclusion it has been satisfied over number of curvilinear models.

IX. REFERENCES

1. Buffa, G. Sangalli, R. V. Azquez, “AN Introduction of Isogeometric Analysis”, Santiago de Compostela, 2010.
2. T.J.R. Hughes, J.A. Cottrell, Y. Bazilevs, “Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement”, Computer Methods Applied Mechanics, Vol. 194, pp. 4135–4195, 2005.
3. Vinh Phu Nguyena, Cosmin Anitescu et al., “Iso-geometric analysis: An overview and computer implementation aspects”, Mathematics and Computers in Simulation.
4. Fubeder D., Simeon B. et al., “Fundamental aspects of shape optimization in the context of Iso-geometric Analysis (IGA)”, Computer Methods in Applied Mechanics and Engineering, Vol.-286, pp.-313 to 331, 2015.
5. Falco C., et al., “GeoPDEs: A research tool for Iso-Geometric Analysis of Partial Differential Equations (PDEs)”, Advances in Engineering Software, Vol.-42, pp. 1020-1034, 2011.
6. Vuong A. V. et al., “ISOGAT: A 2D tutorial MATLAB code for Iso-geometric Analysis (IGA)”, Computer Aided Geometric Design, Vol-27, pp. 644-655, 2011.
7. Xinkang Li et al., “NURBS based IGA of Beams & Plates using Higher Order Sher Deformation Theory (HSDT)” Hindawi Publication, volume 2013.
8. Daniel Rypl, Patzak Borek, “From the finite element analysis to the Iso-geometric analysis in an object oriented computing environment”, Advances in Engineering Software, vol.44, pp. 116-125, 2012.
9. T.J.R. Hughes a, A. Reali b,d,e, G. Sangalli, “Efficient quadrature for NURBS-based isogeometric analysis”, Comput. Methods Appl. Mech. Engrg. Vol. 199, pp. 301–313, 2010.
10. Dennis Ernens, “Finite Element Methods with exact geometry representation including IsoGeometric Analysis, NURBS Enhanced Finite Element Method and Aniso Geometric Analysis” Delft University of Technology.
11. S. S. Bhavikatti, “Finite Element Analysis”, New Age International Publisher, 2005.
12. J. A. Cottrell, T. J. R. Hughes, Y. Bazilevs, “Iso-geometric Analysis: Toward Integration of CAD and FEA”, John Wiley & Sons, Ltd, 2010.