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Theories and Recommendations for Shear Analysis of Concrete with FRP Reinforcement

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Abstract: *Shear capacity evaluation has remain a challenging task due to complexity of shear transfer compared the flexural capacity because of various affecting parameters. The state-of-the-art review is done about the shear mechanism, shear failure patterns and theories related to shear analysis. Various shear models with due assumptions have been developed based on lower-bound plasticity theory which rely on stress redistribution with steel reinforcement. But, lower-bound plasticity theory cannot be applied to the concrete elements with brittle FRP reinforcements. It is summarized that the realistic approach towards the shear capacity with FRP reinforcement which considers the compatibility, equilibrium and the material constitutive laws in combination like crack-based analysis reviewed in the paper.*

Keywords: *Shear Capacity; Fiber Reinforced Polymer (FRP); Equilibrium; Compatibility; Stress Redistribution*

I. THEORIES FOR SHEAR ANALYSIS OF REINFORCED CONCRETE MEMBERS

Cambridge astrophysicist Sir Arthur Eddington has quoted that “No experiment is worthy of credence unless supported by an adequate theory.” The researcher has to see the applicability of the finding after extensive research. Real job of the researcher is that to understand the experimental behaviour of the complex phenomena influenced by various variables and present the findings with justification of adequate theory. Such as in determining the flexural strength of reinforced concrete elements, Hooke’s observation in 1678 that plane sections remain plane is used universally. [1]

Understanding shear in concrete has always challenged researchers. Various theories have been developed to understand the shear behaviour in concrete along with extensive experimental programs, however, is not sufficient to predict the shear-capacity of beam effectively. The reason for having the poor quality design provisions is about 20 variables that influences the shear behaviour as stated by Professor Frits Leonhardt in “Shear and Torsion” at FIP Congress in1970. [1].

A detailed description of how a reinforced concrete beam carries shear is now available [6]. Shear analysis of reinforced concrete elements is performed separately on members with or without shear reinforcement.

Theories based on equilibrium considerations like strut-and-tie models and stress field can be applied when shear reinforcement is provided for safe designs. Theories also considering compatibility conditions and the tensile strength of concrete like compression field-based theories and fixed-angle softened-truss model have also been developed allowing accurate predictions of the shear response of transversely reinforced members.

Shear in members without shear reinforcement has traditionally been estimated by means of purely empirical or semi-empirical expressions like compressive force path method. Modified compression field theory (MCFT) has been applied successfully to the members without shear reinforcement.

Each shear model assumes a different equilibrium-state within the beam; none is based on the actual stress distribution. Despite this, all the theories have been used safely to design steel-reinforced concrete. Each shear model relies on the *lower-bound* (or safe load) *theorem of plasticity* which assumes different equilibrium states and not the actual stress distribution.

II. STRESS-REDISTRIBUTION

Based on simplifying assumptions, postulated equilibrium state is used in lower-bound plasticity theory that balances the external loads; however, it does not satisfy the compatibility. Fig. 1 summarizes equilibrium state for the stress redistribution. The actual stress distribution at the working load may not match the postulated equilibrium state but stress redistribution in ductile material would balance the design load and also satisfy the compatibility. Hence, stress redistribution and ductility are vital if lower-bound plasticity is used in design.

Stress-redistribution cannot occur in brittle reinforcement up to the large extent. The equilibrium state and compatibility requirements cannot be satisfied without stress-redistribution. Hence, lower-bound plasticity theory cannot be applied to the brittle FRP reinforcement.

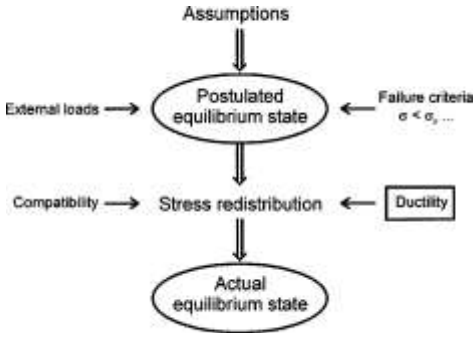

Fig. 1 Stress redistribution and lower bound plasticity theory

III. SHEAR TRANSFER MECHANISMS

The analysis of reinforced concrete beam with ductile or brittle reinforcement must be based on the actual stress-state which satisfies equilibrium and compatibility both and linked with material constitutive laws. Hence, a detailed understanding is required about the mechanism through which the shear is carried by the beam. There are six shear resisting mechanism in the beam as mentioned below are broadly divided into two parts like beams with shear reinforcement termed as "with stirrups" and without shear reinforcement as "without stirrups".[3,4]

- The shear resistance of the uncracked concrete compression zone
- Dowel action of the flexural reinforcement
- Aggregate interlock (Shear Friction) along the crack
- Arching action
- Residual tensile stresses across cracks
- Shear forces carried by shear reinforcement

A four-point simply supported beam would give the clear understanding of different shear mechanisms. The moment carried by a beam can be represented by an internal force couple between the compression-zone concrete and flexural reinforcement actions. For equilibrium in a shear-span, the moment must vary along the beam according to $V=dM/dx$ along the shear-span.

Shear contribution of uncracked concrete depends on the depth of the concrete compressive zone and its strength. The shear contribution increases with deepening of the uncracked concrete zone or strength of concrete increases. Dowel action is the ability of longitudinal reinforcement to transfer shear forces near cracks. The aggregate interlock, also referred to as the shear friction, depends on maximum size of aggregate, compressive strength of concrete and the shear crack width. Wider cracks, smaller aggregate and higher concrete strength are less effective to this mechanism for shear resistance. In arch action vertical component of compressive strut formed in the uncracked concrete transfers the shear near the end of a beam, where the constant horizontal component is reacted by the tensile flexural reinforcement. Arching action is negligible in slender beams, where the shear span to depth ratio is greater than 2.5. Residual tension in cracked concrete only exists for cracks less than 0.15 mm wide. This means that the shear cracks will be able to transfer shear forces until they are larger than 0.15 mm. The shear reinforcement bridges the cracks and transfers shear forces of the crack. Truss analogy provides the understanding of shear carried by shear reinforcement.

IV. COMPATIBILITY IN THE SHEAR-SPAN AND SHEAR FAILURE

Development of the tooth models and shear-compression models for steel-reinforced concrete has necessitated an examination of compatibility requirements in the shear-span of a beam.

4.1 Crack Propagation

Inclined cracks developed in the shear span determine the arch or beam mechanism to carry shear. Compatibility of each component of beam must be considered in conjunction with crack propagation. Two different modes of shear failure describes the manner in which the compression-zone concrete fails

- Shear-compression failure, and
- Diagonal-tension failure.

4.2 Shear-Compression Failure

Shear-Compression failure is often described as “crushing of concrete”. Micro cracks developed in compression zone in absence of triaxial confinement of concrete, coalesce and results in shear compression failure. Confinement reduces the shear action in compression zone; however, it is increased by the shear reinforcement present in the beam.

4.3 Diagonal-Tension Failure

The concrete immediately in front of a crack is subjected to a tension field that causes the crack to propagate diagonally into the beam. In absence of shear-compression failure, the crack propagates towards the point where load is applied. Because of that compatibility is not maintained between the compression zone concrete and flexural reinforcement across the crack, hence the beam action is not possible. An unstable diagonal-tension failure splits the beam into two pieces [6].

4.4 Compatibility of the Flexural Reinforcement

Compatibility of the flexural reinforcement with the concrete is also responsible for shear failure of concrete in compression-zone where it crosses a crack (Fig. 2). The local crack opening has both axial and shear components with respect to the reinforcement at the base of shear crack. Two modes that achieve the compatibility of reinforcement across a crack are stretching of the unbonded reinforcement and slip of the bonded reinforcement [7].Generally in steel, the slip is assumed to be negligible compare to plastic stretching. While in FRP, elastic stretching and slip both are important. Force in the flexural reinforcement depends on the bond characteristics, the stiffness of the reinforcement, and the unbounded length of reinforcement for a given crack opening

4.5 Reinforcement-Concrete Bond

Reinforcement-concrete bond is key factor in shear failure [4,5]. Reinforcement can pull out from concrete in weaker beam, which destroys the beam action necessary for load transfer [6].

4.6 Unbonded Length of Reinforcement

The unbounded length of reinforcement can be larger than the crack width. It is increased by local cracking developed due to load transfer across the reinforcement-concrete interface [7]. For a given crack width, an increase in the unbonded length results in a reduction in the reinforcement strain, and hence the load carried by the reinforcement. This leads to re-establish the equilibrium of the beam section and crack must propagate further into the compression-zone. Sometimes, it may not be possible to re-establish equilibrium, which results in either shear-compression or diagonal-tension failure of the compression-zone of concrete, hence the failure of the beam.

4.7 Dowel-Splitting

The load in shear carried by dowel action by the reinforcement across the crack is negligible in case of steel reinforcement, while it is even lower with FRP reinforcement because of low transverse stiffness. As described in the preceding section, dowel action can cause longitudinal cracking of the concrete along the flexural reinforcement even if the load is smaller [6]. Dowel splitting results in increase of unbonded length of flexural reinforcement, which can lead to unstable crack propagation into the compression-zone and results in failure of the beam [8].

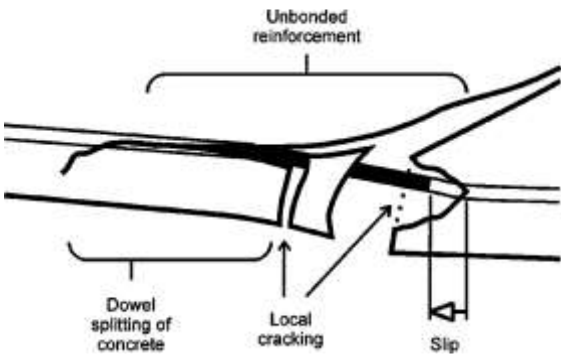

Fig. 2 Compatibility of flexural reinforcement with crack opening

4.8 Dowel Rupture

Dowel rupture does not occur with steel reinforcement but because of low transverse strength of FRPs make them rupture. The dowel rupture is the mode of failure due to dowel action and is the combined effect of shear and tensile actions in reinforcement before its pure tensile strength is achieved.

V. PREDICTING THE SHEAR FAILURE LOAD

Ideally, prediction of shear-capacity of a beam can be done by detailed examination of the shear transfer mechanisms, crack propagation, and failure of the beam components. The code recommendations are based on the first crack load, which are considerably lower than ultimate load specifically in short shear span [6]. In case of beam without stirrups, shear capacity is found empirically in design codes (BS8110 1985; Eurocode 2 1992; ACI 1999). While the beams with stirrups has not been examined much in detail as that in beams without stirrups.

5.1 Estimation of shear contribution

Total shear capacity of a reinforced concrete beam is assumed to be the superposition of the "concrete contribution" (V_C) and "stirrup contribution" predicted using the45° truss analogy (VS) (BS81101985; Eurocode 2 1992; ACI 1999):

$$V = V_C + V_S$$

The "concrete contribution" is the shear capacity of an equivalent beam without stirrups considering that the curvature at failure is concentrated at a single critical crack. The "stirrup contribution" assumes continuous curvature along the shear-span [3]. Thus, the equilibrium-state postulated by superposition is not compatible, and requires stress redistribution. Even how the stirrups are embedded in the surrounding concrete is not considered.

In case of brittle FRP reinforcement stress-redistribution does not take place, hence, the "concrete" and "stirrup contributions" cannot be superposed. The truss analogy does not consider compatibility of the reinforcement across a crack which cannot predict dowel rupture of the brittle flexural reinforcement.

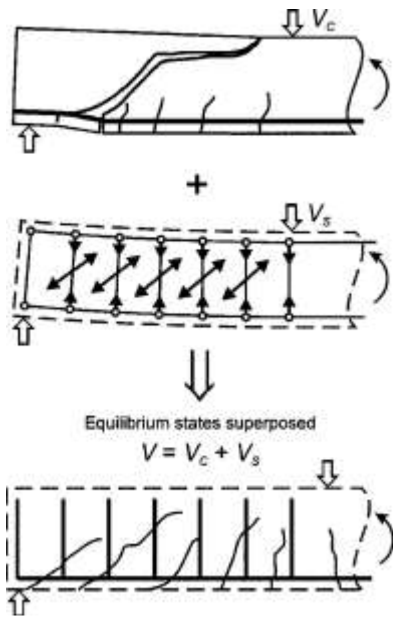

Fig. 3 Superposition of ''concrete'' and ''stirrup contributions'' using 45° truss analogy

5.2 Truss Analogies

The truss analogy proposed by RITTER-MÖRSCH (1899) for the calculation of shear reinforcement in reinforced concrete members is the oldest and well known example of reasoning by means of "strut-and-tie models". The assumed internal equilibrium-state comprises inclined compressive struts of concrete and tensile shear reinforcement. A 45° strut angle is used to predict failure load when the shear reinforcement yields in original truss *analogy* [9]. While varying compressive strut angle is used to give reinforcement yield and web concrete failure simultaneously in *modified truss analogy using explicit plasticity theory* [10]. Both the truss analogies rely on stress-redistribution from the postulated fully developed plastic truss, to the actual equilibrium-state [2]. The truss mechanism is not observed experimentally: The assumed compressive struts would have to cross curved cracks in the shear-span, even though the crack surfaces are completely separate. Furthermore, the truss analogies are sectional design methods. The shear capacity is calculated on a critical vertical section, whereas in reality failure occurs along a single crack [6].

5.3 Compressive-Force-Path Method

The Compressive Force Path Concept is based on realistic assessment of shear capacity of a beam without stirrups that assumes shear failure by excessive tensile stresses perpendicular to the compressive path but remains empirical. This happens due to change in the direction of the force path requiring a tensile resultant, dilation in the vertical direction due to varying intensity of the compressive stress field, and high tensile stresses at the tip of cracks. It should be noted that in this concept the entire shear is carried above the neutral axis through assumed stress conditions in the compression zone and shear reinforcement is placed to restrict critical shear crack which is assumed to yield. Finally the net shear capacity is found by superposition of concrete and shear reinforcement contribution.

5.4 Compression-Field Theory

The shear capacity is related to the compression in diagonally cracked concrete through equilibrium. Compression-field theory is based on the biaxial response of square elements of steel-reinforced concrete. The original constitutive relationships were derived analytically calculating the actual angle θ, and the crushing strength of the concrete struts. The crushing strength is a function of the tensile strain perpendicular to the strut, but these have been replaced by more realistic empirical equations [11]. If an isolated element is considered, the internal equilibrium state is avoided by the use of empirical constitutive relationships. If the element is part of a beam, simplifications must be made that to rely on stress-redistribution. Moreover, compression-field theory is based on sectional design method similar to truss analogy; however, shear is not a sectional failure.

5.5 Modified Compression Field Theory (MCFT)

The MCFT provides more realistic assessment of shear capacity of wide range of beam with shear reinforcement and also for the cases for without shear reinforcement. The compression field theory gives conservative estimate of shear strength because of neglecting tensile stress contribution in cracked concrete. While, MCFT accounts the contribution of tensile stresses in the concrete between cracks. The most authoritative assumption in the MCFT model is to treat the cracked concrete as a new material with empirically defined stress-strain behaviour in reinforced concrete which can be traditionally different from the stress-strain curve of uncracked concrete.

5.6 Simplified Compression Field Theory (SCFT)

The shear strength of a section is a function of the two parameters inclination θ of the diagonal compressive stresses in the web, and the factor for tensile stresses in the cracked concrete, β, both depend on the longitudinal straining of the web, ε_x. For members without transverse reinforcement, β and θ values calculated from the MCFT are given as functions of ε_x and the crack spacing s_{xe} in a table, however, many engineers prefer simple equations to tables because they give a continuous range of values and are more convenient for spread sheet calculations [12], introduced the concept of simplified compression field theory for the shear design of concrete beams. The method provides a simplified version of MCFT, which provides simple equations for β and θ to be determined from the basic expressions of the MCFT, where the calculation of full load deformation analysis is not needed.

5.7 Strut and Tie Model (STM)

The STM is most rational and simple design approach which is widely used for the members like deep beams and/or non flexural members. The term truss is an assemblage of pin joined, uni-axially stressed tension or compression member where term D is used for Disturbed or D-region and term B is used for Beam or B-region, where in B-region beam action and in D-region arch action is expected. This design approach is not unique due to specific geometry of the structures, significant re-distribution of the stresses after cracking and is based on lower bond theory of plasticity and efficiency factors are applied to uni-axial strength of concrete to account for concrete softening. Sufficient reinforcement needs to be provided in all the direction of RC structure for the ductile failure. Application of Strut and Tie Model for more slender beams without transverse reinforcement may lead to unsafe solution. Diagonal crushing strength of concrete is required to be reduced [13] in case of slender beams without transverse reinforcement. In an equilibrium model, the designer specifies at least one load path which ensures that no part of this path is overstressed.

VI. SHEAR PREDICTION USING FRP

6.1 Concrete Contribution

The “concrete contribution” in shear design of FRP reinforced element is modified by the ratio of the stiffness of FRP to steel in the proposed recommendations. Certainly, the stiffness of the reinforcement affects the shear capacity of the beam; however, it is one of the influencing parameters when steel is substituted by FRP reinforcement. Furthermore, the suggested design proposals of “concrete contribution” with FRP reinforcement have been validated with the beams *without* shear reinforcement experimentally. The load carried by the concrete at failure is governed by compatibility of the cracked concrete with the shear reinforcement in a shear reinforced beam. It is convenient to split the shear capacity of a beam conceptually into a “concrete contribution” and “stirrup contribution”; however, two mechanisms cannot be treated in isolation with brittle reinforcement; they must be compatible. The code proposals do not recognize this [2].

6.2 Stirrup Contribution

Shear reinforcement must not fail at a large crack opening; however, it must be effective at small crack openings, to restrain crack propagation. The "stirrup contribution" is the shear carried by stirrups across the crack just before the failure of first stirrup. The lower-bound plasticity theory allows us to assume that all the reinforcement yields along a crack with steel reinforcement. In contrast, the strain in FRP shear reinforcement varies along a crack [2]. Compatibility condition determines the distribution of stirrup strain along a crack; however, compatibility of the shear reinforcement depends upon the bond characteristics of the reinforcement with surrounding concrete. Different FRP bars are manufactured with different surface finishes, and have different bond characteristics. Hence, the load carried by stirrups in the beams with the shear reinforcement of same ultimate strain-capacity but different bond characteristicswill differ. The code proposals for FRP reinforcement assumes an artificial stirrup yield strain the "allowable strain" for use in the truss analogy [14].

6.3 Crack Based Analysis

The writers have described an analytical investigation of compatibility requirements in the region of a shear crack [7]. The cracked section must remain in equilibrium with applied external load. The crack-based analysis examines the propagation of crack into the beam and determines the variation in compatibility variables that satisfies equilibrium. Compatibility conditions are described by the horizontal and vertical projected lengths of the crack, and the crack opening angle across the cracked section. The flexural reinforcement, shear reinforcement, and concrete must remain compatible with the crack geometry. Stretching of the unbonded reinforcement pull out of the reinforcement from the surrounding concrete and the response of the compression-zone concrete constitutes the equilibrium conditions across the cracked section.

Thus, for shear design, the brittle FRP reinforcement is modelled by an imaginary pseudoplastic FRP reinforcement, which is elastoplastic with a yield strain equal to the allowable strain. The crack-based analysis can be used to examine the effect of assuming pseudo-plastic FRP reinforcement [7]; however, it does not predict individual stirrup failure. The original intention of "allowable strain" concept was to limit the stirrup strain so that the crack width at failure remains similar to that in steel-reinforced concrete which would allow fully developed "concrete contribution". The ''allowable strain'' concept does not consider compatibility of the shear reinforcement with the cracked concrete, which the crack-based analysis shows is essential for shear design. There is no reason to suppose that a uniform limiting strain can be applied to find the net shear carried across a crack.

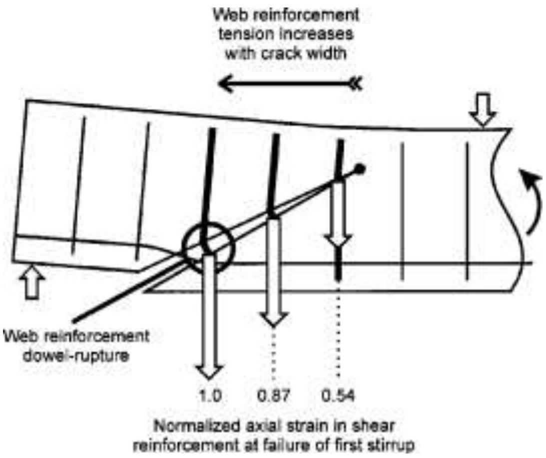

Fig. 4 Variation in shear reinforcement strain and concrete reinforcement slip along shear crack, just before failure of first stirrup

VII. EXPERIMENTAL EVALUATIONS

Muttoni and Ruiz [3] have studied the shear-carrying mechanisms after the development of critical shear crack. To estimate the shear strength of the members without shear reinforcement, a model is proposed based on an estimate of the crack width in the critical shear region, taking also into account the roughness of the crack and the compressive strength of concrete. In the study important observation is made that the plasticity-based solutions with an inclined compression strut overestimate the shear strength when a critical shear crack develops inside the theoretical strut and they defined the mechanism of shear resistance after development of critical shear crack. Developed analytical expression shown good agreement with 285 test results.

Robert Machial and co-authors [15] have calculated the shear strength contribution of FRP transverse reinforcement of112 beams without stirrups and 54 beams with FRP stirrups from the published literature and compared the performance of ACI 440.1R-06 and JSCE 1997 guidelines. The JSCE 1997 guideline had better performance for calculating the concrete shear strength than the ACI guideline with an average calculated over experimental shear strength of 1.41 compared to 1.92. However, the ACI guideline outperformed the JSCE 1997 guideline for calculating the transverse shear strength with an average calculated shear over experimental shear of 1.69 compared to 2.26.

Ehab A. Ahmed and co-authors [16] have tested3 CFRP reinforced and one steel reinforced large scale concrete beams with a length of 7,000 mm and a T-shaped cross section with variable stirrup's material and shear reinforcement ratio. The test results were compared to predictions provided by different codes and design guidelines. The current ACI 440.1R-06 design method provides conservative predictions; however, the CAN/CSA S6-06 and JSCE 1997 underestimate the contribution of the FRP stirrups due to low strain limits. It was observed that concrete contribution enhances after first shear crack also closer spacing of CFRP stirrups enhances the shear resistance due to confinement and controls the shear crack. Additionally it is observed that using the 4,000 microstrain as FRP stirrup strain at the ultimate limit state enables predicting the shear strength of concrete members reinforced with FRP stirrups with adequate conservatism level.

A.K. El-Sayed and co-authors [17] have proposed the modification in shear design equation proposed by ACI Committee 440. Test results of the beam reinforced with different types and ratios of FRP bars indicates that the proposed a design approach for evaluating the concrete contribution to the shear resistance of FRP-reinforced concrete beams that accounts for the axial stiffness of FRP longitudinal reinforcement underestimates the concrete shear strength. The proposed equation was verified against experimental shear strengths of 98 specimens tested to date, and the calculated values are shown to compare well. In addition, the proposed equation was compared to the major design provisions using the available test results. Better and consistent predictions were obtained using the proposed equation.

Mohsen Kobraei and co-authors [18] have investigated the effects of CFRP bars as shear reinforcement instead of steel stirrups in RC beams. All beams were cast using a high strength concrete (HSC) and self-compacting concrete (SCC). In this approach, results of seven laboratory specimens show that using carbon fibre reinforced polymer (CFRP) shear reinforcement can be an acceptable alternative for normal stirrups in RC beams. It is observed that the CFRP shear reinforcement bars can be considered as an attractive alternative instead of normal stirrups where their ρ are 50 to 85% ρ_b. The beams with FRP reinforcement have greater capacity with less deflection compared to the concrete beams with steel reinforcement. Even, the ultimate capacity of both observed without significant difference with high strength concrete. The crack development with CFRP has good extension throughout the beam; however, the width of cracks is larger than the beams with steel reinforcement. In case of larger spacing of the replacement CFRP shear reinforcement bars, the number of cracks increase but the width of the crack is narrower.

Jongsung SIM and co-authors [19] have tested a set of 12 beams with steel and GFRP reinforcement. It is observed that with increase in shear span to depth ratio (a/d) failure load and shear failure angles decreased and even dominant failure mode change from shear failure mode to flexural and shear failure mode. Shear strengthening efficiency of the GFRP stirrups observed is similar to the steel stirrups, however, this very fundamental study for characterization of the shear failure, the authors are encouraged to extend it to further research on the failure characterization of beams reinforced with the newly developed GFRP stirrups.

Emile Shehata and co-authors [20] have tested and analyzed fifty two designed panel specimens and ten large scale T-section along with 118 beams tested by others. Important evaluations are proposed in form of design guidelines for the use of FRP as shear reinforcement in concrete structures. It is observed that the bend effect on the strength capacity of FRP stirrups is more critical than the kink effect which is responsible for the development of beam action and contribution of FRP stirrup in shear contribution. Shear deformations are affected by elastic modulus and bond characteristics of stirrups material. Wider shear cracks, smaller depth of the compression zone and poor dowel action associated with the use of FRP as longitudinal reinforcement leads the less concrete contribution of the beams reinforced with CFRP strand in flexure. A minimum shear reinforcement ratio is recommended to ensure that the shear strength exceeds that shear cracking load for concrete beams with FRP reinforcement. To achieve the at least 50% guaranteed strength of FRP stirrups bend radius, r_b, should not be less than larger of four times the effective bar diameter or 50 mm and tail length, l_d, six times the effective bar diameter or 70 mm. Limiting strain of 0.002 is recommended for both CFRP and GFRP stirrups to control the shear crack width in concrete beams.

VIII. CONCLUSIVE SUMMARY

Noted literature warns about use of lower-bound theory for shear design with FRP reinforcement; however, proposed design clauses reflect their steel reinforced origins, and rely on truss analogies. Both designers and researcher must recognize that the assumption like in shear analysis contributions of stirrups and concrete can be superimposed is not safe with FRP reinforcement, when the assumption or simplification of the postulated equilibrium state is made. Large scale redistribution of stresses is not possible as per truss analogies with FRP reinforcement; however, small size redistribution may be possible by with necessary simplified assumption.

A realistic crack-based model for shear in FRP reinforced concrete based on a fundamental examination of equilibrium, compatibility, and the material constitutive laws in a beam is more valid approach than current design proposals which considers compatibility requirement in detail can be extended for the shear analysis.

Consistent effort is made by the researcher to develop the confidence over the use of FRP as an internal reinforcement. Extensive experimentation is done in with options without and with shear reinforcement using FRP reinforcement with reference to the design recommendations published by the various organizations like ACI, CSA, and JSCE. It is unique recommendation that the experimental and analytical results are varying and modification in the design recommendations which conservatively considers the special characteristics of FRP reinforcement in place of steel reinforcement is required.

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