



Magnetic Flux Compression by Expanding Plasma Sphere

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Abstract — A conceptual study of magnetic flux compression inside an electrical conductor by an expanding inertial fusion plasma sphere across the magnetic field is performed numerically using a two dimensional magneto-hydrodynamic (MHD) simulations. The concept may find application in inertial fusion energy (IFE) system as a direct energy conversion scheme to convert a part of fusion plasma kinetic energy into pulsed electrical energy. This paper will report the details of the numerical algorithm developed to study this phenomena. Preliminary theoretical analysis is given for the analysis of MHD interchange instabilities of expanding plasma across magnetic field. Overall efficiency of the system is determined numerically for a typical set of initial plasma and system parameters. The efficiency of the proposed system is found to be greater than 50 % for a typical set of initial parameters used.

Keywords- Magnetic Flux, Magneto Hydrodynamic (MHD) Simulation, Inertial Fusion Energy (IFE), Plasma

I. INTRODUCTION

The process of magnetic flux compression (MFC) inside an electrically conducting surface by an expanding diamagnetic inertial fusion plasma sphere (from fusion micro-explosions) and its application as a direct energy conversion scheme to convert a part of plasma kinetic energy into pulsed electrical energy is reported in our recent work¹. The basic idea is to use a conducting surface that encloses expanding diamagnetic plasma across external magnetic field. This field will be excluded by the diamagnetic plasma by the currents produced in the plasma. The inductive electromotive force (EMF) will induce currents in the shielding conductor and thus converts a part of plasma kinetic energy into pulsed electrical energy. It is observed that during the final stage of MFC, the plasma dynamics becomes distorted due to non-uniform deceleration caused by magnetic pressure and the Rayleigh-Taylor (RT) instability begins to evolve near plasma-vacuum interface. Also, an elongation of plasma blob in the axial direction is observed. These effects lead to a non-spherical expansion of the plasma and cause large deformations to the Lagrangian mesh. Therefore, a magneto-hydrodynamic (MHD) algorithm using an arbitrary Lagrangian Eulerian (ALE) formulation is developed to overcome the limitations of pure Lagrangian computation¹.

II. BASIC CONCEPTS AND DESCRIPTION

Figure 1 shows a schematic representation of magnetic flux compression inside an electrically conducting shield (a cylindrical coil in the figure) by an expanding plasma sphere. We assume a spherical plasma is created at the center of the shielding conductor, e.g. from fusion micro-explosions. In [1] a cylindrical coil is used as a shielding conductor, where a single coil is used for both pickup and to produce initial field. The shielding conductor can also be a cylindrical shell with a number of axially and radially distributed pickup loops inside it to extract electrical energy [7]-[16].

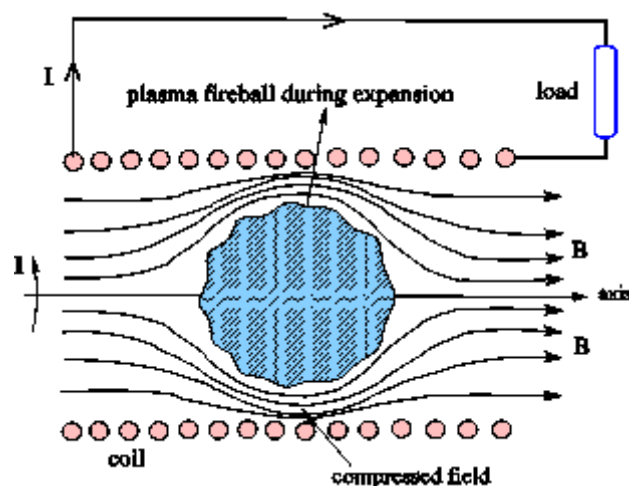


Figure 1. Schematic showing magnetic flux compression by expanding plasma (no to scale)

The expanding plasma does work against the magnetic field and loss energy. The increase in the magnetic energy due to magnetic flux compression is accompanied by an increase in the conductor current to conserve the magnetic flux. In other words a part of plasma kinetic energy can be converted in to electrical energy by the compression of magnetic flux between the conductor and plasma.

III. INITIAL CONDITIONS

The initial conditions were taken from earlier published data for D-3He fusion plasma. Although the required ignition energy is substantially higher for D-3He, the reaction products will consist predominantly of charged particles, which can be electromagnetically manipulated. In D-3He fusion products about 80 % of the fusion energy is carried by 14-MeV protons. Immediately after the fusion, the plasma is in a state of extremely high temperature (greater than few tens of keV) and density ($\sim 10^6 \text{ kg/m}^3$) with a radius of 150-250 μm . Therefore, initially, it undergoes free expansion in the applied initial magnetic field. Considering this fact, we start our simulation with an initial plasma radius of about 1.0 cm with a coil of radius 1.5. We have taken the coil axial length as approximately 2-4 times the radius. For the present study, a uniform plasma density and energy profile is assumed. The initial plasma kinetic energy is assumed to be 140 MJ with plasma mass equal to 6 mg. The initial magnetic field is varied from 2-10 Tesla, which is considerably higher than the magnetic field values used in earlier reported works.

IV. EQUATIONS OF MHD MODEL

The following MHD equations are to be solved.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad (4.1)$$

$$\rho \left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla p + \nabla \cdot [\vec{T} + \vec{R}] \quad (4.2)$$

$$\frac{\partial}{\partial t} \left(\rho \vec{I} + \frac{\rho \vec{u}^2}{2} \right) + \nabla \cdot \left[\rho \vec{u} \left(\vec{I} + \frac{\vec{u}^2}{2} \right) + p \vec{u} - \vec{R} \cdot \vec{u} \right] = -\vec{J} \cdot \vec{E} \quad (4.3)$$

Here, the notations in the above equations have usual meanings. The magnetic vector potential in the plasma is governed by

$$\frac{\partial \vec{A}}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 \vec{A} + \vec{u} \times \nabla \times \vec{A} \quad (4.4)$$

In the free space, the vector potential satisfies the following equation.

$$\nabla^2 \vec{A} = 0 \quad (4.5)$$

The field components are calculated from the vector potential as given below:

$$\vec{B} = \nabla \times \vec{A} \quad (4.6)$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} \quad (4.7)$$

V. COMPUTATIONAL METHOD

For MFC problems with imperfectly known boundary conditions, complex geometries and materials having large variations in electrical conductivity (materials from insulator to conductor); the finite difference time-domain (FDTD) method for electro-magnetics [2]-[5] may be the best suited method to calculate the amplified field in the compression volume and the diffused of magnetic flux into the confining conductors. Detailed discussions on the advantages and disadvantages of such approach are given in [2]-[5] (also, see Refs. therein). However, for the present geometry setup only the theta-component of current does exist in both the confining conductor and plasma. Similarly, the boundary conditions are either known explicitly or can be specified in terms of theta-component of current. In addition to that, all the non-zero field components (B_r , B_z and E_θ) can be calculated from the theta-component of vector potential, A_θ . Therefore, the MHD algorithm for the present case can be formulated using A_θ . It has an advantage that only the theta-component of vector potential is need to be transported instead of two magnetic field components. Therefore the algorithm is computationally less expensive. Also, it may produce less divergence of B compared to the case where two components of magnetic field are transported.

In the present algorithm, the magnetic field diffusion into the plasma sphere is solved implicitly using magnetic vector potential. The advection terms are computed using a second-order monotonic upwind scheme (MUSCL) due to van Leer (a non-linear total variation diminishing (TVD) limiter). The MUSCL scheme used here is less diffusive compared to the Lax-Wendroff and FCT scheme. We have compared the results obtained using our advection method and other common methods FCT and Lax-wendroff. An iteration procedure, using a second-order-accurate alternating direction implicit (ADI) scheme, is used to calculate the field components in free space with the help of known boundary conditions at plasma and conducting surfaces. The deceleration of plasma sphere due to Lorentz force is calculated using Maxwell stress tensor. The proposed algorithm is formulated using a 'dimensionally-split' approach to extent the calculations into two dimensional cases. Similarly, an 'operator-split' approach is used with three distinct phases in each direction (axial and radial): electromagnetic diffusion, Lagrangian motion, and Eulerian advection or remap. To avoid the

unnecessary material diffusion caused by Eulerian methods, the interface between plasma and vacuum or low density material is explicitly tracked by using volume-of-fluid (VOF) method [6].

A fixed Eulerian mesh is used with a virtual Lagrangian calculation. The effect of Lagrangian deformations are remapped on to the original mesh during Eulerian advection and remap step. We have used a spatially staggered mesh, where velocities are defined at cell faces and density, internal energy, pressure, etc. are defined at the mid-point of the cell, see Figure 2.

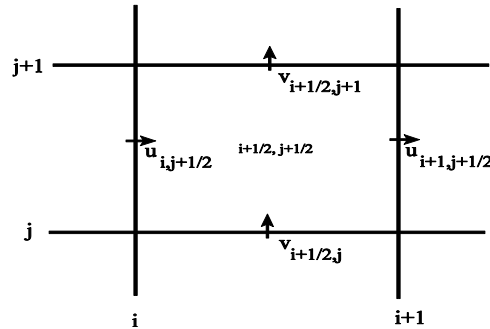


Figure 2. Typical staggered mesh used in the computation

The physical variables are updated using a 'dimensionally split' method to extent the calculations into two dimensional case. Similarly, the algorithm is formulated using an 'operator-split' approach with three distinct phases in each direction (axial and radial): electromagnetic diffusion, Lagrangian motion, and Eulerian advection or remap. In each time-step, first a Lagrangian calculation is performed in one direction (say radial) followed by a radial remap step. After these set of calculations, a Lagrangian step is performed in the other direction (axial) followed by an axial remap step. The calculation of electromagnetic diffusion is performed in the Lagrangian phase calculation for each direction. To avoid the biasing effects the sequence of steps in radial and axial direction is alternated during subsequent time-steps.

VI. RESULTS AND DISCUSSIONS

The initial conditions used in the simulation are described in earlier sections. Figure 3 shows the variation of the magnetic vector potential--representing the magnetic flux compression inside the compression volume. The magnetic field outside the plasma is amplified by the flux compression, while the field inside the plasma is getting dilated. As expected the maximum field between the coil and the plasma is observed at an axial point $z=0$.

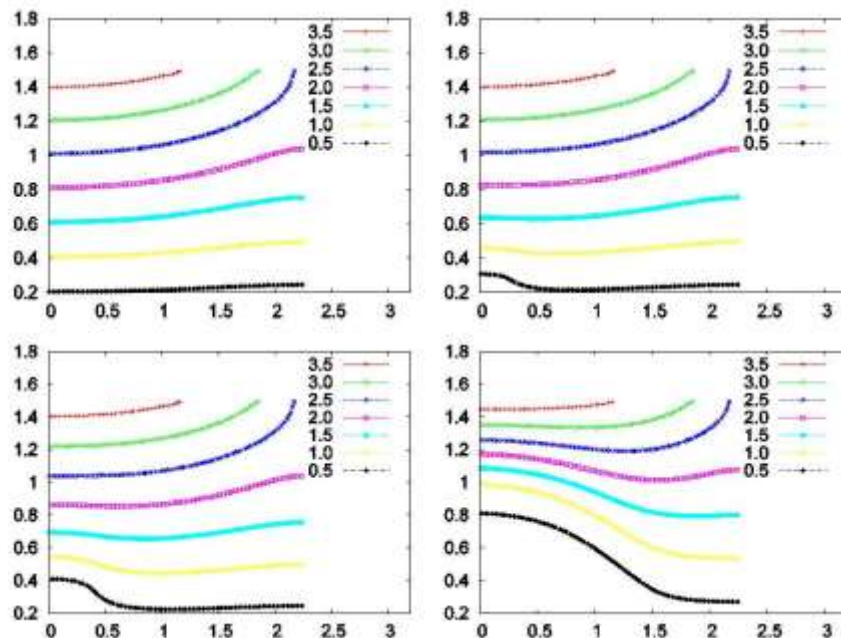


Figure 3. Spatial variation of magnetic vector potential shows magnetic flux compression inside the compression volume and field dilation inside the plasma.

Figure 4 shows the evolution plasma density at different times during the compression phase. Due to high initial plasma beta value the early stage of plasma expansion is less affected by the magnetic field. The expansion perpendicular to the field direction is decelerated by the magnetic pressure while the expansion along the field lines is essentially unaffected. The plasma forms nearly a shell like structure near the stagnation point, see Figure 4.

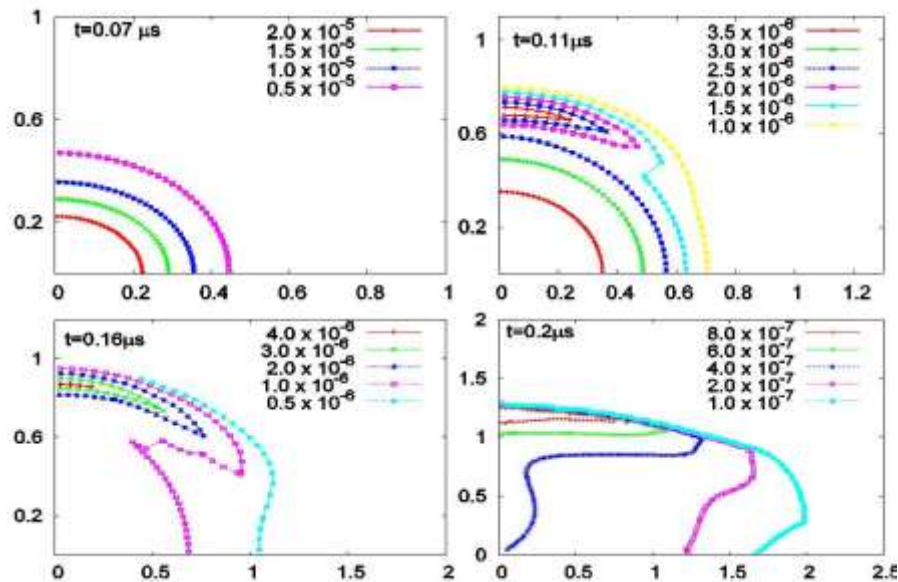


Figure 4. Plasma density variation at different times during the expansion phase.

The overall system efficiency calculated is found to be 56%. To study the variation of efficiency with different load conditions, we have varied the load inductance and the efficiency is determined in each case. These are summarized in Table 1. As the inductance increase the efficiency increases and tend to saturate.

Sr. No.	Load (mH)	Input Energy (MJ)	Load Energy (MJ)	Efficiency
1	0.05	407	8	5.5
2	0.1	421	16	10
3	0.3	478	41	26
4	0.5	534	63	38
5	0.7	590	79	47
6	1.0	675	97	56

Table 1. Overall system efficiency with different inductive load

Next, we have examined the variation of plasma stopping radius at $z=0$ location for different initial magnetic fields. The coil radius, length and number of turns are 3 m, 9 m and 30 respectively. The magnetic field is varied from 1.4 to 6.3 T. The simulation results are compared with simple analytical solution, see Figure 5.

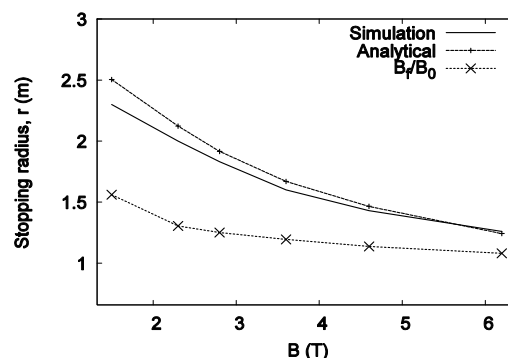


Figure 5. Simulation results are compared with simple analytical solution

A reasonable agreement is found except for the cases with low initial magnetic fields. The difference in results especially for the cases with low magnetic fields is due to the fact that the analytical expression neglects the effect of field amplification and therefore over-estimates the stopping radius. This effect will be higher for the cases with low initial magnetic fields, where the radial plasma expansion will be higher and likely to have considerable difference between initial and final magnetic fields. Another reason for the difference in simulation results and analytical result is

due to the assumption of uniform expansion of plasma sphere while deriving the expression for stopping radius. It is seen that the plasma sphere expands non-uniformly (plasma blob expansion is higher along the axial direction) such that considerable energy spread occurs along the axial direction. This will under-estimate the stopping radius at $z=0$ location as the particles are likely to expand freely along the axial direction.

VII. CONCLUSIONS

A conceptual study of magnetic flux compression inside a cylindrical coil by an expanding fusion plasma sphere has been performed numerically using two-dimensional MHD simulations. Preliminary theoretical analysis shows that no MHD interchange instabilities may grow during the first expansion phase of the plasma for a typical set of system parameters used. The concept can be used as a method to convert a part of fusion plasma kinetic energy into pulsed electrical energy. An overall system efficiency of $\sim 56\%$ obtained for a typical system with appropriate load conditions. Approximately 78% of plasma kinetic energy is converted into electrical energy with appropriate inductive load conditions. The simulation results indicate that the proposed system is promising in terms of overall efficiency.

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