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Time dependence of Gravitational Constant and Deceleration Parameter in the Framework of Brans-Dicke Theory

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Abstract

A simple empirical expression has been chosen to represent the Brans-Dicke (BD) scalar field ϕ as a function of the scale factor in such a manner that the density of matter (ρ) of the universe has the same functional dependence upon the scale factor. An interaction and a consequent inter-conversion between the matter and dark energy have been considered in this formulation. The present model is based on the generalized BD theory where the BD parameter ω is regarded as a function of the scalar field ϕ. Solving the field equations for a spatially flat Robertson-Walker space-time, the functional forms of $a(t)$, $q(t)$, $H(t)$, $G(t)$ have been determined and their inter-dependence has been analyzed in detail. The parameter $\omega(\varphi)$ has been found to have a negative value. The possibility of an inter-conversion between dark energy and matter has been taken into account by introducing a slowly varying function $f(t)$. A signature flip of deceleration parameter, implying a transition from deceleration to acceleration, and an increase of gravitational constant with time, have been found in the present study. Analytically and also through numerical plots, the time dependence and inter-dependence of the relevant parameters have been explored.

Keywords: *Brans-Dicke theory; Accelerated expansion of the universe; Gravitational constant; Dark energy; Brans-Dicke scalar field; Signature flip of deceleration parameter*

Introduction

On the basis of some recent observations regarding the expansion of the universe, it is known that the universe has undergone a smooth transition from a decelerated to an accelerated phase of expansion [1,2]. The normal matter has a positive definite density and pressure and gravitates in the usual manner. It is inferred that there must be some other kind of matter responsible for the observed acceleration, which makes the effective pressure sufficiently negative and gives rise to a repulsive effect. The matter of this kind is referred to as dark energy. A great endeavour has been made in the past to analyze the nature of it. A large number of models have been proposed to account for the accelerated expansion.

Cosmological constant Λ is the simplest choice for the dark energy [3]. CDM model has a serious drawback in connection to the value of cosmological constant Λ. The currently observed value of Cosmological constant Λ for an accelerating Universe does not match with that of the value in Planck scale or Electroweak scale [4]. The problem can be rendered less acute if one tries to construct dark energy models with a time dependent cosmological parameter. Many such models have been proposed but they have their own problems [5, 6].

A suitable alternative to the dynamical Λ models are the scalar field models in which the equation of state of dark energy changes with time. Among the many proposed scalar field models, quintessence models are the ones endowed with a potential so that the contribution to the pressure sector, $p = {}^1\!/_2\, \dot{\varphi}^2 - V(\varphi)$, can evolve to attain an adequately large negative value, thus generating the observed cosmic acceleration [7, 8]. One main drawback of these quintessence models is that most of the quintessence potentials are chosen arbitrarily and do not have a proper theoretical justification explaining their genesis. Naturally a large number of other alternative scalar field models, for example the tachyon [9, 10], k-essence [11, 12], holographic [13, 14] dark energy models have appeared in the literature with their own virtues and shortcomings.

In most of the scalar field models the cold dark matter and dark energy are normally allowed to evolve independently. However, there are attempts to include an interaction amongst them so that one grows at the expense of the other [15]. Non minimal coupling of the scalar field with the dark matter sector through an interference term in the action has helped in explaining the cosmic acceleration. Such fields are called 'Chameleon fields' and they have proved to be useful in playing the role of dark energy candidates [16, 17].

Non minimal coupling between the scalar field and geometry, especially in the frame work of Brans-Dicke theory, also pose themselves as possible candidates for explaining the observed acceleration. Modification of the Brans-Dicke (BD) theory by adding a potential $V(\varphi)$, which is a function of the BD scalar field φ itself, can serve as models explaining the acceleration of the Universe [18].

BD theory of cosmology has been analyzed with the aid of different models. To name a few, Sheykhi et al. [19] worked with the power-law entropy-corrected version of BD theory defined by a scalar field and a coupling function. In another literature Sheykhi et al. [20] considered the HDE model in BD theory to think about the BD scalar field as a possible candidate for producing cosmic acceleration without invoking auxiliary fields or exotic matter considering the logarithmic correction to the entropy. Jamil et. al. [21] studied the cosmic evolution in Brans-Dicke chameleon cosmology. Pasqua and Khomenko [22] studied the interacting logarithmic entropy-corrected HDE model in BD cosmology with IR cut-off given by the average radius of the Ricci scalar curvature.

Some models have also been suggested where a quintessence scalar field introduced in the BD theory can give rise to a late time acceleration for a wide range of potentials [23]. An interaction between dark matter and the BD scalar field showed that the matter dominated era can have a transition from a decelerated to an accelerated expansion without any additional potential [24]. On the other hand BD scalar field alone can also drive the acceleration without any quintessence matter or any interaction between BD field and dark matter [25].

However, the problem with many of these models is that the matter dominated Universe has an ever accelerating expansion contrary to the observations. Besides this, in order to explain the recent acceleration many of the models require a very low value of the BD parameter ω of the order of unity whereas the local astronomical experiments demand a very high value of ω [26].

Our present study is based on a generalized form of Brans-Dicke theory where, instead of choosing a constant value for the BD parameter, we have a variable BD parameter $\omega(\varphi)$ which is a function of scalar field parameter (φ). We have taken into account an inter-conversion between matter and dark energy which is mainly responsible for the accelerated expansion of the universe.

Theoretical Model

For a spatially flat Robertson-Walker space-time, the field equations in the generalized Brans-Dicke theory are [27]

$$3\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{\varphi} + \frac{\omega(\varphi)}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - 3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} , \quad (1)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{\omega(\varphi)}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - 2\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} - \frac{\ddot{\varphi}}{\varphi} . \quad (2)$$

Combining (1) and (2) one gets,

$$2\frac{\ddot{a}}{a} + 4\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{\varphi} - 5\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} - \frac{\ddot{\varphi}}{\varphi} . \quad (3)$$

Considering the conservation of matter of the universe we propose the following relation.

$$\rho = f(t)(\rho_0 a_0^3)a^{-3} = f(t)\rho_0 a^{-3}, \qquad (a_0 = 1) \quad (4)$$

Here a_0 and ρ_0 are the scale factor and the matter density of the universe respectively at the present time. According to some studies, the matter content of the universe may not remain proportional to $\rho_0 a_0^3$ [31]. There may be an inter-conversion between dark energy and matter (both baryonic and dark matter) [34]. In the present model, a factor $f(t)$ has been introduced to account for the conversion of matter into dark energy or its reverse process. It is assumed here that this conversion, if there is any, is extremely slow. This assumption of slowness is based on the fact that there are studies where the variation of density of matter is expressed as $\rho = \rho_0 a^{-3}$, which actually indicates a conservation of the total matter content of the universe [27]. In the present calculations, the factor $f(t)$ is taken as a very slowly varying function of time, in comparison with the scale factor. Equation (4) makes it necessary that $f(t) = 1$ at $t = t_0$ where t_0 denotes the present instant of time when the scale factor $a = a_0 = 1$ and the density $\rho = \rho_0$.

To make the differential equation (3) tractable, let us propose the following ansatz.

$$\varphi = \varphi_0 a^{-3}$$
(5)

Here φ has been so chosen that it has the same dependence upon scale factor as that of the matter density. This choice of φ makes the first term on the right hand side of equation (3) independent of the scale factor (a).

In equation (5) we have taken $\varphi = \varphi_0$ for $a = a_0 = 1$.

Combining (3) and (5) and treating f as a constant we have,

$$\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = -f\frac{\rho_0}{\varphi_0}$$
(6)

In terms of Hubble parameter $H = \frac{\dot{a}}{a}$, equation (6) takes the following form.

$$\left(\dot{H} + H^2\right) - H^2 = \frac{dH}{dt} = -f\frac{\rho_0}{\varphi_0}$$
(7)

Integrating equation (7) and taking $H = H_0$ at $a = a_0 = 1$,

$$H = \frac{\dot{a}}{a} = f\frac{\rho_0}{\varphi_0}(t_0 - t) + H_0$$
(8)

Integrating (8) and requiring that $a = a_0 = 1$ at $t = t_0$,

$$a = Exp\left[-\frac{1}{2}f\frac{\rho_0}{\varphi_0}(t^2 + {t_0}^2) + \left(f\frac{\rho_0}{\varphi_0}t_0 + H_0\right)t - H_0 t_0\right] \qquad (9)$$

In deriving the equations (8) and (9), f has been treated as a constant assuming its extremely slow time variation compared to the scale factor. The time dependence of f is determined later in this study and incorporated in equation (9). Figure 1 shows the variation of scale factor as a function of scaled time (t/t_0) where $t_0(= 14\ billion\ years)$ is the age of the universe. The time dependence of Hubble parameter has been shown in figure 2.

Using (9), the deceleration parameter $q = -\frac{\ddot{a}a}{\dot{a}^2}$ becomes

$$q = -1 + \frac{f\rho_0/\varphi_0}{\left(\frac{f\rho_0}{\varphi_0}(t_0 - t) + H_0\right)^2} .$$
(10)

Now letting $q = q_0$ at $t = t_0$ in (10), one obtains $q_0 = -1 + \frac{\rho_0}{H_0^2\varphi_0} = -0.9652$.

Its negative sign shows that the universe is presently passing through a state of accelerated expansion and this fact is consistent with other studies.

Equation (10) clearly shows that a signature flip in q takes place at $t = \tau$ where,

$$\tau = t_0 - \left(\sqrt{\frac{\varphi_0}{f\rho_0}} - H_0\frac{\varphi_0}{f\rho_0}\right)$$
(11)

Taking $\tau = \alpha t_0$ with $\alpha < 1$ we get the following quadratic equation.

$$H_0 x^2 - x + t_0(1 - \alpha) = 0 \quad \text{with} \quad x = \sqrt{\frac{\varphi_0}{f\rho_0}}$$
(11a)

To have a single solution of this equation we must have $\alpha = 1 - \frac{1}{4H_0 t_0} = 0.757$ (11b)

Thus we get, $x \equiv \sqrt{\frac{\varphi_0}{f\rho_0}} = \frac{1}{2H_0}$ leading to $f = \frac{4H_0^2\varphi_0}{\rho_0}$
(11c)

The values of different cosmological parameters used in this study are,
$H_0 = 72\left(\frac{Km}{Sec}\right) per\ Mega\ Parsec = 2.33 \times 10^{-18} sec^{-1}$, $t_0 = 14\ billion\ years = 4.415 \times 10^{17} sec$, $\varphi_0 = \frac{1}{G} = 1.498 \times 10^{10} m^{-3} Kg s^2$, $\rho_0 = 2.831 \times 10^{-27} Kg/m^3$ (present density of dark matter + ordinary matter).

Let us now formulate the factor $f(t)$ from different criteria to be satisfied by it.

Based on (11b, 11c), we may write,

$$f(t) = 4\frac{H_0^2\varphi_0}{\rho_0} \quad \text{at } at\ t = \alpha t_0 = t_0 - \frac{1}{4H_0}$$ (11d)

According to an initial requirement we had,

$$f(t) = 1\ at\ t = t_0$$ (11e)

Let us now propose a relation between f and t which will satisfy the conditions expressed by (11d) and (11e). This relation is given by,

$$f = \left(\frac{1}{f_2}\right)^{\frac{t/t_0-1}{1-\alpha}} \quad \text{with} \quad f_2 = 4\frac{H_0^2\varphi_0}{\rho_0}$$ (11f)

This functional form of $f(t)$ keeps it positive which is a requirement of equation (4), since density of matter can not be negative. This time dependent form of $f(t)$ has been in all expressions in the present study.

Incorporating equation (11f) into equation (10), the deceleration parameter has been plotted in figure 3 and it clearly shows a signature flip at $t = \alpha t_0 = 0.757t_0$. It shows that a very small duration of decelerated expansion is preceded and followed by accelerated expansion. Figure 4 shows the variation of matter content ρa^3, as a function of time, relative to its value at $t = 0.5t_0$. It clearly shows a decrease of matter content with time, indicating a conversion of matter into dark energy, which is responsible of accelerated expansion.

According to Brans-Dicke theory, the gravitational constant is the reciprocal of the scalar field parameter φ. Therefore, using equations (5) and (9) we have,

$$G = \frac{1}{\varphi} = \frac{a^3}{\varphi_0} = \frac{1}{\varphi_0}Exp\left[-\frac{3}{2}\frac{f\rho_0}{\varphi_0}(t^2 + {t_0}^2) + 3H_0(t - t_0) + 3\frac{f\rho_0 t_0}{\varphi_0}t\right]$$ (12)

and the fractional change of G per unit time is given by,

$$\frac{\dot{G}}{G} = \frac{1}{G}\ \frac{d}{dt}\left[\frac{1}{\varphi_0}Exp\left\{-\frac{3}{2}\frac{f\rho_0}{\varphi_0}(t^2 + {t_0}^2) + 3H_0(t - t_0) + 3\frac{f\rho_0 t_0}{\varphi_0}t\right\}\right] \quad \text{with } f = \left(\frac{1}{f_2}\right)^{\frac{t/t_0-1}{1-\alpha}}$$ (13)

According to Brans-Dicke theory, $G = \frac{1}{\varphi}$. Using this relation and equation (5) we get, $\left(\frac{\dot{G}}{G}\right)_{t=t_0} = 3H_0 = 2.2 \times 10^{-10}\ Yr^{-1}$.

According to a study by Weinberg, $\left(\frac{\dot{G}}{G}\right)_{t=t_0} \leq 4 \times 10^{-10}\ Yr^{-1}$ [33]. Our result is consistent with this observation.

In the figures (5) and (6), we have plotted the time variation of G and $\frac{\dot{G}}{G}$ respectively. The gravitational constant is found to increase with time with a varying rate. Both curves show that the universe is presently passing through a stage where the rate of G variation is the smallest. This increasing nature of G has been found in some other studies [28, 29, 30, 32].

At $t = t_0$, $\frac{\dot{G}}{G}$ is positive, implying that the gravitational constant is presently increasing with time. The condition for having $\frac{\dot{G}}{G} > 0$ is

$$t < t_0 + \frac{H_0\varphi_0}{\rho_0} = 1.277 \times 10^{19} = 28.925\ t_0 \quad .$$ (14)

The gravitational constant will be decreasing with time and consequently $\frac{\dot{G}}{G}$ will be negative for $t \geq 28.925\ t_0$. It implies that beyond 28.925 times the present age of the universe, the gravitational constant will be decreasing with time. Using (2) and (5) we get,

$$\omega(\varphi) = -\frac{2}{3}\left(1 + \frac{\ddot{\varphi}\varphi}{\dot{\varphi}^2}\right) = -\frac{2}{9}\left(7 - \frac{\ddot{a}a}{\dot{a}^2}\right) = -\frac{2}{9}(7 + q).$$ (15)

Equation (15) shows that the Brans-Dicke parameter $\omega(\varphi)$ has a linear relationship with the deceleration parameter (q).

At $t = t_0$ we have,

$$\omega(\varphi_0) = -\frac{2}{9}(7 + q_0) = -1.341 \quad .$$ (16)

Substituting for q in equation (15) from equation (10)

$$\omega(\varphi) = -\frac{2}{9}(7 + q) = -\frac{2}{9}\left[6 + \frac{f\rho_0/\varphi_0}{\left[\frac{f\rho_0}{\varphi_0}(t_0 - t) + H_0\right]^2}\right] \quad .$$ (17)

Equation (17) shows the time variation of Brans-Dicke parameter $\omega(\varphi)$.

Combining the equations (5) and (9) one gets,

$$\varphi = \varphi_0 a^{-3} = \varphi_0 Exp\left[-3\left(-\frac{1}{2}\frac{f\rho_0 t^2}{\varphi_0} + \left(H_0 + \frac{f\rho_0 t_0}{\varphi_0}\right)t - H_0 t_0 - \frac{1}{2}\frac{f\rho_0 {t_0}^2}{\varphi_0}\right)\right] \quad (18)$$

Figures (7) and (8) show the variation of the Brans-Dicke parameter $\omega(\varphi)$ as a function of scaled time and the scalar field φ respectively. It is found to be negative over the entire range of study. It is found that the most negative value of ω corresponds to the time of signature flip of deceleration parameter.

Figures (9) and (10) show the variation of the deceleration parameter as functions of scale factor and ω respectively. The first one shows that the transition from deceleration to acceleration took place in the past at $a = 0.726$, its present value being unity. The second graph shows that more negative values of ω drives the universe toward a state of deceleration.

Figures (11) and (12) show the effect of gravitational constant on the scale factor and deceleration parameter respectively. The scale factor increases with a gradually decreasing slope. The deceleration parameter rises sharply to a positive value and then falls to a value very close to minus one.

Conclusions

The present model clearly shows that a generalized scalar tensor theory, where the BD parameter ω is regarded as a function of the scalar field φ, can account for an accelerated expansion of the universe at the present time. We have assumed, in the present study, an empirical dependence of the BD scalar field parameter φ on the scale factor (a). The results of this study show that the universe has made a transition from a decelerated phase of expansion to the present accelerated phase and it will continue to remain in the state of acceleration. This study also shows that there was accelerated expansion in the very early stage of the universe before the beginning of the deceleration phase. These calculations reveal that the gravitational constant increases with time. The rate of this increase is consistent with other studies in this regard. The present study shows the variation of the BD parameter $\omega(\varphi)$ graphically as a function of time and also the scalar field parameter φ. To take into account the exchange of energy between the field of matter (both dark and baryonic) and dark energy we have introduced a function $f(t)$ in this model. The time dependence of this function has been determined empirically by using the information regarding its values at two different instants of time. It has been shown graphically that there has been a decrease of matter content with time, indicating a conversion of matter into other forms of energy responsible for the accelerated expansion of the universe. This model can be improved further by changing the ansatz representing the dependence of the scalar field φ upon the scale factor (a). Instead of formulating the time dependence for f, this study can also be carried out by making a suitable choice of an ansatz, representing the dependence of the function f upon the scale factor and then, by incorporating this function in the initial differential equation whose solution gives the time dependence of the scale factor.

FIGURES

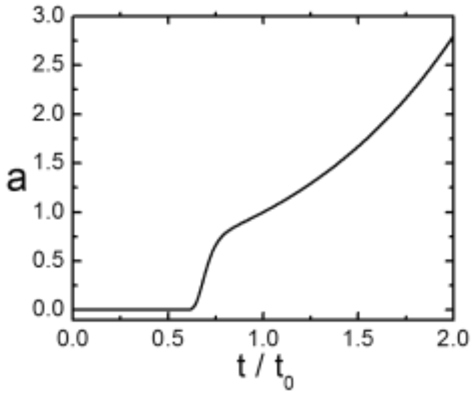

Figure 1. Variation of scale factor (a) as a function of time.

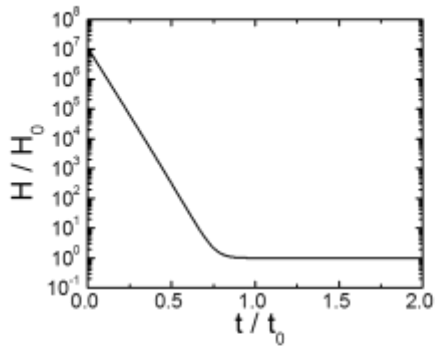

Figure 2. Variation of Hubble parameter as a function of time.

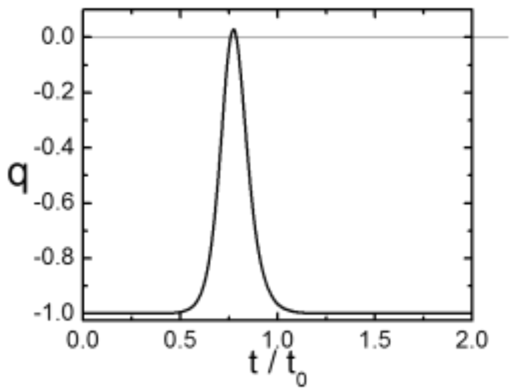

Figure 3. Variation of deceleration (q) parameter as a function of time.

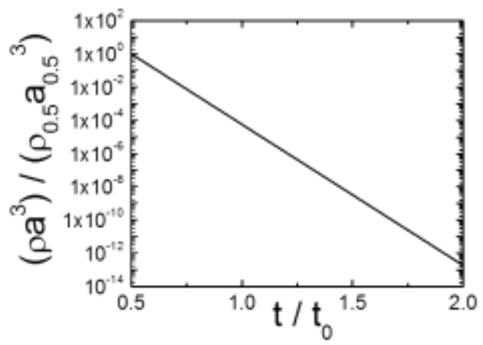

Figure 4. Variation of matter content, as a function of time, relative to its value at $t = 0.5t_0$

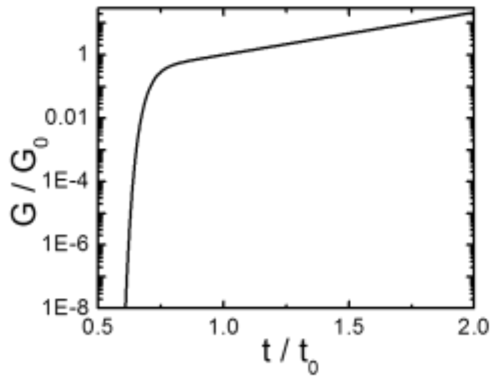

Figure 5. Variation of gravitational constant as a function of time.

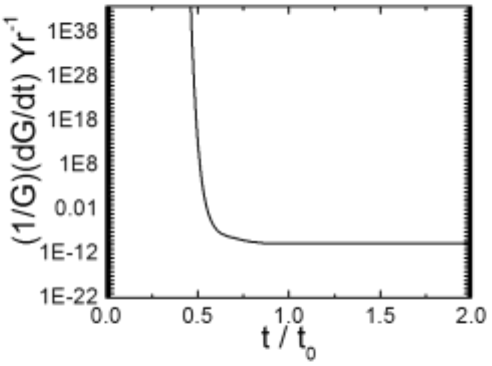

Figure 6. Variation of fractional change of G per second, as a function of time.

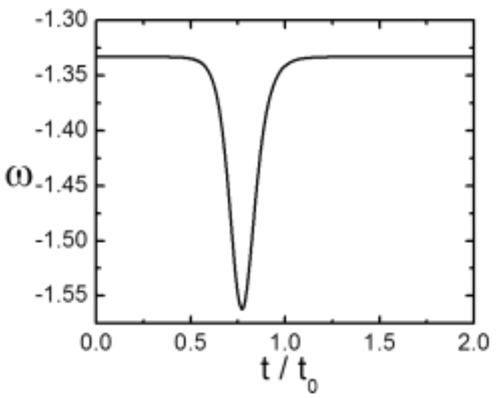

Figure 7. Variation of $\omega(\varphi)$ as a function of time.

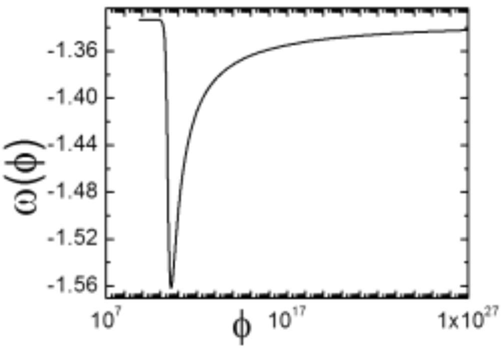

Figure 8. Variation of $\omega(\varphi)$ as a function of the scalar field φ.

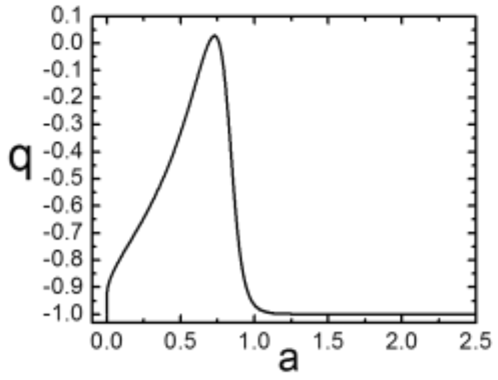

Figure 9. Variation of deceleration parameter (q) as a function of scale factor.

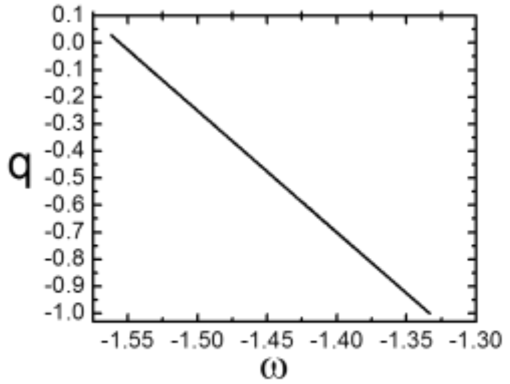

Figure 10. Variation of deceleration parameter (q) as a function of ω.

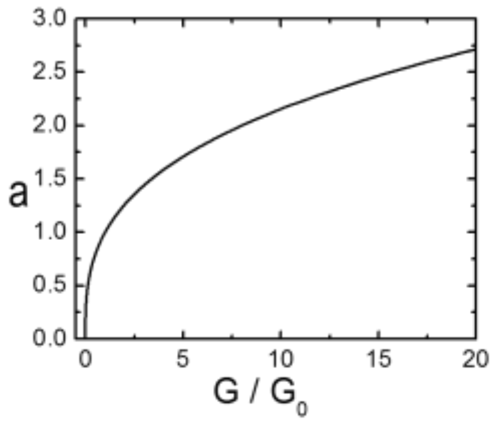

Figure 11. Variation of scale factor (a) as a function of gravitational constant.

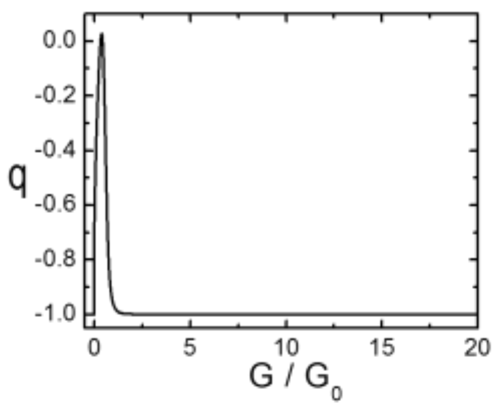

Figure 12. Variation of deceleration parameter (q) as a function of gravitational constant.

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