

Simulation and Analysis of conventional based Power System State Estimation using MATLAB

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Abstract — As the power system grows larger and more complex, real time monitoring and control becomes vital in order to achieve a reliable operation of the power system. Especially, with world wide restructuring of the power industry, power system state estimation has got more importance as a real time monitoring tool. Because of the open access and the operation of the transmission networks, the patterns of power flow in deregulated power system have become less predictable compared to the integrated systems of the past. In order to achieve a more secure and economic operation of such a complicated system, utility operators are to be properly informed of the operating condition of the current power system. Hence real time monitoring of system is an extremely important part of the modern day Energy management system or energy control centers.

In this paper, a new fundamental WLS based Static State Estimation technique namely Singular Value Decomposition is used which is far better than WLS based Normal Equation method. Furthermore, comparative analysis of both WLS based techniques has been taken out for various operating situations of the power system. For the verification of both the algorithms IEEE-14 bus test case system is utilized and results are carried out using m.file in MATLAB software.

Keywords - State Estimation, Weighted Least Square (WLS), Normal Equation Method, Singular Value Decomposition (SVD), MATLAB software

I. Introduction

First of all, the concept was introduced by Fred Schweppe, who proposed the idea of State Estimation in power systems in 1970s. State Estimation is, by definition, the mathematical procedure that estimates the states (bus voltages and phase angles) from the network data and sensor information [1]. Now days, State Estimation has become one of the most vital key component for Energy Management System (EMS). State estimation is essential for monitoring, controlling and optimization of a power system and it behaves like a core of the power system. In other words, the one can say state estimation acts like a filter between the raw measurements received from the system and all the application functions that are required the most reliable database for the current state of the system [2],[3]. In the general sense, the static state estimation is defined as: "If the state vector is obtained for an instant of time "k" from the measurement set of the same instant of time, then such estimation is called static state estimation". Static state estimation is widely used

in power system and plays a very important role for the reliable operation of the transmission and distribution [4].

There are many papers that refer measurement error, communication error and topology error identification in state estimation, especially the Weighted Least Square case. The most popular static estimation method in industry is weighted least squares. Comprehensive treatment of modern power system state estimation can be found in books like Ali Abur and Gomez Exposito in 2004. Usually a system is designed to be observable prior to the state estimation for most operational conditions. Temporary unobservability may still occur due to unexpected network topology changes or failures in the telecommunication systems. In the conventional state estimation methods based on WLS method using the normal equation approach may fail to provide solution when gain matrix is ill-conditioned due to temporary unobservability and if it does not have re-observability analysis [5].

The SVD is a very powerful set of techniques for dealing with sets of equations or matrices that are either singular or

else numerically very close to singular. In the case of over-determined system, the SVD provides a solution that is the best approximation in the WLS sense. In the case of under-determined system, the SVD provides a solution whose values are the smallest in the WLS sense. In the over-determined or completely-determined case, the singularity from the normal equations implies what is known as an unobservable system. In the case of under-determined case, the singularity implies that there is no unique solution to the problem. However, the SVD will provide a particular solution and a null space vector or equivalent close to zero vectors for each singularity [6].

II. WLS based Normal Equation Approach [5]

The vital task of any state Estimator is to generate the best possible state estimates from the measured data which is corrupted with measurement noise or error. Consider the set of Measurements given by the vector z :

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} h_1(x_1, x_2, \dots, x_n) \\ h_2(x_1, x_2, \dots, x_n) \\ \vdots \\ h_m(x_1, x_2, \dots, x_n) \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} = h(x) + e \dots(1)$$

Where, z and x are the vectors of measurements and state variables respectively. $h(x)$ is the measurement matrix, and e is the measurement noise vector, which is assumed to be made of independent random variables with Gaussian distribution.

The WLS estimate is, therefore, the vector x that minimizes the weighted sum of the squares of the residuals ($r = z - h(x)$) between the actual measurements and estimated levels, i.e.

$$\text{Minimize } J(x) = (z - h(x))^T R^{-1} (z - h(x)) \dots\dots\dots(2)$$

Where, R^{-1} is the inverse of the covariance matrix.

Matrix R is diagonal and contains the covariance of the measurements (if they are known). This permits applying higher weighting to measurements that are known to be more accurate. R is replaced by the identity matrix if the same instrumentation is used to obtain them. The solution to Eq. (1) in the WLS sense is obtained by solving the following equation:

$$(H^T R^{-1} H) x = (H^T R^{-1}) z \dots\dots\dots(3)$$

Real and reactive power measurements are used in the conventional state estimation, the measurement equation is

non-linear. In such case, the solution for Eq. (2) must be obtained through an iterative algorithm. The flow chart for this algorithm is shown in Fig: 1.

III. WLS based Singular Value Decomposition Approach [6],[7]

The SVD is a method for identifying and ordering the dimensions along which data points exhibit the most variation. As SVD is capable of identify where most variation is, it is possible to find the best approximation of the original data points using fewer dimensions.

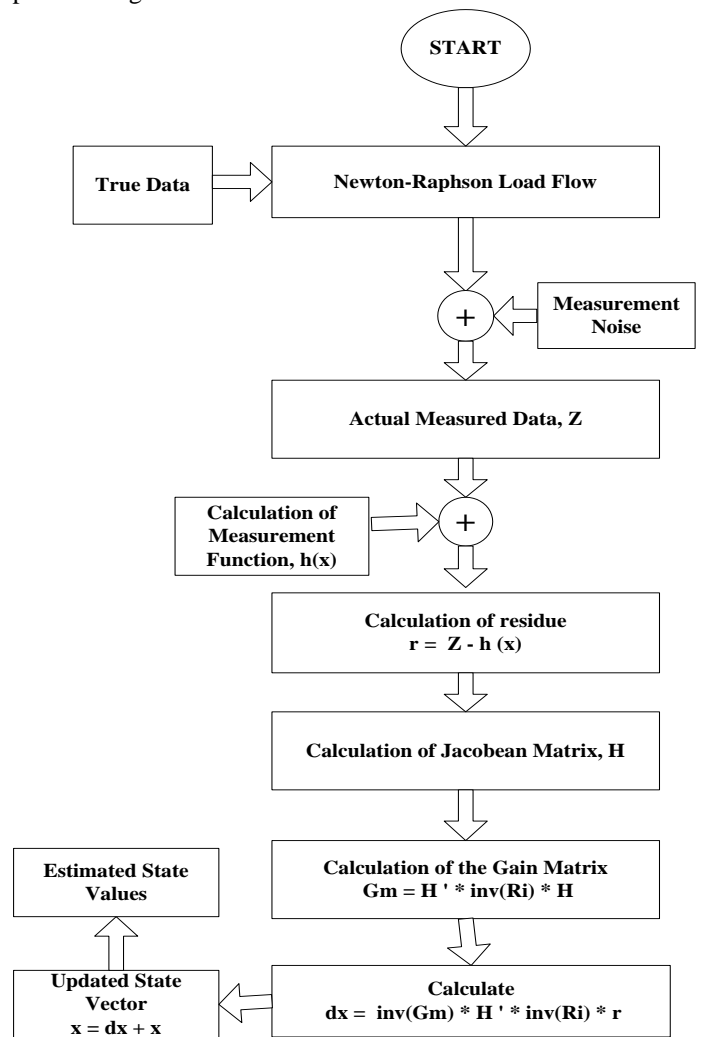


Fig: 1 Flowchart for WLS based Normal Equation Algorithm

The SVD method represents the matrix H ($m \times n$) of Eq. (1) as the product of three matrices [9]. When m is the number of measurement placement, and n is the number of state variable.

$$H = U S V^T \dots\dots\dots(4)$$

W is a diagonal matrix ($n \times n$) with positive or zero elements, which are the singular values of H . Matrices U and V^T are orthogonal matrices, U being a column orthogonal ($m \times n$) matrix and V^T is the transpose of a ($n \times n$) orthogonal matrix.

From, Equations (1) and (4) the following expression of x is obtained:

$$X = V S^{-1} U^T z \dots\dots\dots(5)$$

The flow chart for of the WLS based SVD algorithm is shown in Fig: 2

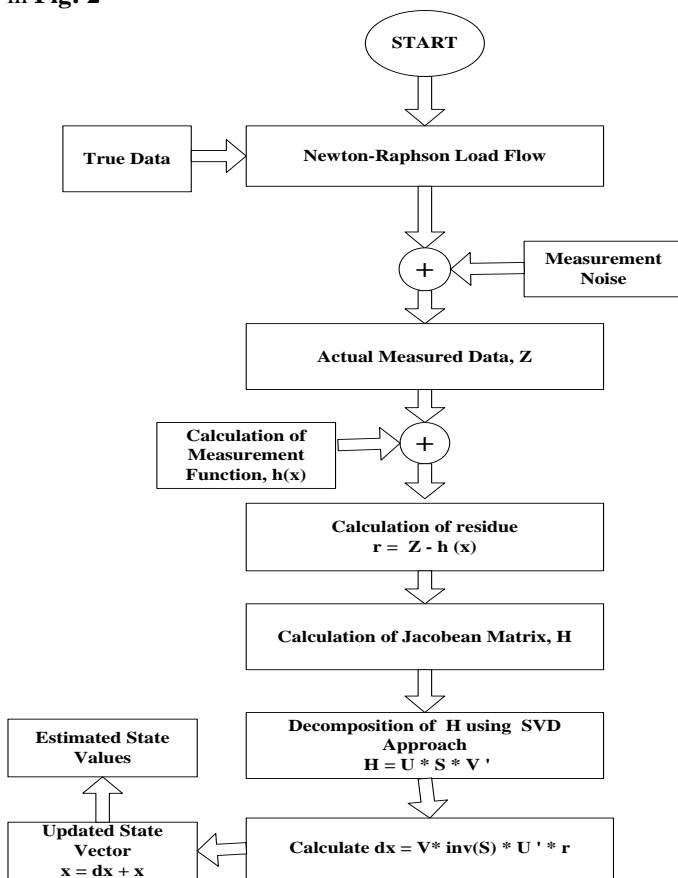


Fig: 2 Flowchart for WLS based Singular Value Decomposition Algorithm

IV. Test System and Simulation Results

The IEEE 14-bus test case system [8] is shown in Fig: 3 is chosen to verify the algorithms and compare the performance of the SVD approach with the normal equation approach in Non-linear WLS power system state estimation. It is to be noted that the initial measurement data set is generated from the Newton-Raphson load flow analysis method. Bus power

injections and line power flows with some voltage magnitudes are taken as the measurement data set. The performance comparison is made for following test cases.

- Case 1: All the measurements are correct (no gross errors).
- Case 2: Gross errors in the measurement data.
- Case 3: Topological errors in the measurement data.

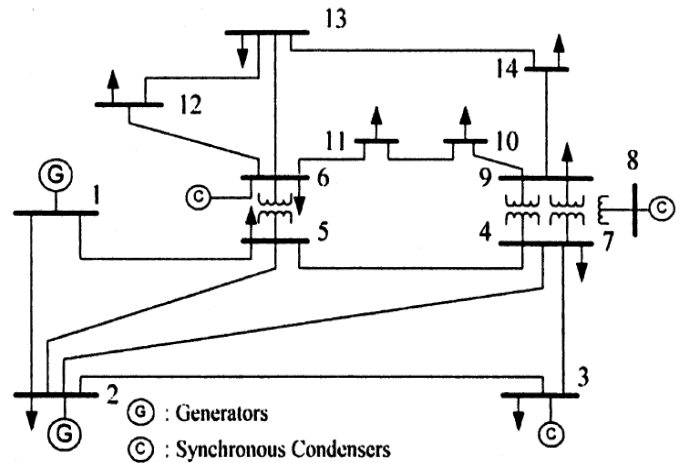


Fig: 3 Single Line Diagram of IEEE-14 Bus Test Case System

Case 1: All the measurements are correct (no gross error).

The absolute Error, Mean Average Error (MAE) and the Maximum Error (MAX) are also compared for both the conventional estimators in these tables. It is observed that under case-1, where all the measurements are correct, having the same performance. Both the estimators are giving the satisfactory results. It is to be noted for the no measurement error the value of Error Matrix R is taken as the one or unity matrix in both of the estimators. TABLE-1 and TABLE-2 shows the estimated voltage magnitudes (V) and bus angle for such a case where no gross errors were present.

TABLE-1: Voltage Estimates for case-1

Bus No.	V (Act.)	V (Con.1)	V (Con.2)	Abs Error (Con.1)	Abs Error (Con.2)
1	1.060	1.0600	1.0600	0.0000	0.0000
2	1.045	1.0026	1.0384	0.0424	0.0066
3	1.010	0.9868	1.0231	0.0232	0.0131
4	1.019	0.9500	0.9873	0.0690	0.0317
5	1.020	0.9541	0.9910	0.0659	0.0290
6	1.070	0.9579	0.9947	0.1121	0.0753
7	1.062	1.0145	1.0501	0.0475	0.0119
8	1.090	0.9874	1.0235	0.1026	0.0665
9	1.056	1.0237	1.0583	0.0323	0.0023
10	1.051	0.9719	1.0086	0.0791	0.0424
11	1.057	0.9743	1.0105	0.0827	0.0465
12	1.055	0.9915	1.0274	0.0635	0.0276
13	1.050	0.9979	1.0336	0.0521	0.0164
14	1.036	0.9906	1.0268	0.0454	0.0092
MAE				0.0584	0.0270
Max.				0.1121	0.0753

Where, **Act.**-Indicates the Actual value of complex bus voltage which is obtained from load flow data. **Con.1** shows Estimated values of complex bus voltage obtained from WLS based Normal Equation method. **Con.2** shows the estimated values of complex bus voltage obtained from WLS based Singular Value Decomposition method.

TABLE-2: Bus Angles Estimates for case-1

Bus No.	Angle (Act.)	Angle (Con.1)	Angle (Con.2)	Abs Error (Con.1)	Abs Error (Con.2)
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	-0.0871	-0.0975	-0.0908	0.0104	0.0037
3	-0.2225	-0.2503	-0.2327	0.0278	0.0102
4	-0.1788	-0.2008	-0.1865	0.0220	0.0072
5	-0.1529	-0.1713	-0.1592	0.0184	0.0063
6	-0.2521	-0.2763	-0.2517	0.0242	0.0004
7	-0.2310	-0.2626	-0.2434	0.0316	0.0124
8	-0.2310	-0.2670	-0.2479	0.0360	0.0169
9	-0.2587	-0.2802	-0.2616	0.0215	0.0029
10	-0.2624	-0.2836	-0.2644	0.0212	0.0020
11	-0.2593	-0.2821	-0.2625	0.0228	0.0032
12	-0.2670	-0.2928	-0.2722	0.0258	0.0052
13	-0.2676	-0.2926	-0.2761	0.0250	0.0085
14	-0.2805	-0.3015	-0.2809	0.0210	0.0004
MAE				0.0220	0.0057
Max.				0.0360	0.0169

Case 2: Gross Errors in the Measurement Data.

Gaussian (Normal) noise of $1e-4$ (0.0001) and $64e-6$ (0.000064) was introduced in the actual bus power measurements (real and reactive) and line measurements (real and reactive) respectively. **TABLE-3** and **TABLE-4** show the estimates for (V) and bus angle for test **case-2**.

TABLE-3: Voltage Estimates for case-2

Bus No.	V (Act.)	V (Con.1)	V (Con.2)	Abs Error (Con.1)	Abs Error (Con.2)
1	1.060	1.0600	1.0600	0.0000	0.0000
2	1.045	0.8218	0.9996	0.2232	0.0454
3	1.010	0.8032	0.9839	0.2068	0.0261
4	1.019	0.7951	0.9814	0.2239	0.0376
5	1.020	0.7739	0.9576	0.2461	0.0624
6	1.070	0.7776	0.9603	0.2924	0.1097
7	1.062	0.8416	1.0175	0.2204	0.0445
8	1.090	0.8106	0.9897	0.2794	0.1003
9	1.056	0.8438	1.0177	0.2122	0.0383
10	1.051	0.7934	0.9756	0.2576	0.0754
11	1.057	0.7968	0.9777	0.2602	0.0793
12	1.055	0.8172	0.9948	0.2378	0.0602
13	1.050	0.8249	1.0011	0.2251	0.0489
14	1.036	0.8153	0.9938	0.2207	0.0422
MAE				0.2218	0.0550
Max.				0.2924	0.1097

The gross errors were introduced randomly in four measurements, namely in real power injection, reactive power injection, real power line flows and reactive power line flow measurements. It is observed from the table that the gross errors present in the measurement data can deteriorate the performance of the WLS based Normal Equations state estimator and have to be removed or reweighed and are to be re-estimated by going through state re-estimation. However, the proposed SVD state estimator is robust in such cases. The bad data can easily be detected in case of proposed method, as there is no data spreading possible unlike conventional state estimation.

TABLE-4: Bus Angles Estimates for case-2

Bus No.	Angle (Act.)	Angle (Con.1)	Angle (Con.2)	Abs Error (Con.1)	Abs Error (Con.2)
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	-0.0871	-0.1308	-0.0867	0.0437	0.0004
3	-0.2225	-0.2548	-0.1545	0.0323	0.0680
4	-0.1788	-0.2679	-0.1812	0.0891	0.0024
5	-0.1529	-0.2265	-0.1556	0.0736	0.0027
6	-0.2521	-0.3798	-0.2641	0.1277	0.0120
7	-0.2310	-0.3695	-0.2557	0.1385	0.0247
8	-0.2310	-0.3778	-0.2633	0.1468	0.0323
9	-0.2587	-0.3952	-0.2755	0.1365	0.0168
10	-0.2624	-0.4000	-0.2786	0.1376	0.0162
11	-0.2593	-0.3934	-0.2737	0.1341	0.0144
12	-0.2670	-0.4047	-0.2806	0.1377	0.0136
13	-0.2676	-0.4051	-0.2811	0.1375	0.0135
14	-0.2805	-0.4249	-0.2954	0.1444	0.0149
MAE				0.1057	0.0166
Max.				0.1468	0.0680

Case 3: Topological Errors in the Measurement Data.

For **case-3** a topological error was simulated as inclusion error of line 2-4. The line was actually out but the status (measurement) showed it to be in the system. Both WLS programs were run with the line flow measurement as zero (both real and reactive), as acceptable. Results for this case are depicted in **TABLE-5** and **TABLE-6**.

TABLE-5: Voltage Estimates for **case-3**

Bus No.	V (Act.)	V (Con.1)	V (Con.2)	Abs Error (Con.1)	Abs Error (Con.2)
1	1.060	1.060	1.0600	0.0000	0.0000
2	1.045	0.9953	1.0347	0.0497	0.0103
3	1.010	0.9794	1.0194	0.0306	0.0094
4	1.019	0.9440	0.9847	0.0750	0.0343
5	1.020	0.9448	0.9898	0.0752	0.0302
6	1.070	0.9533	0.9932	0.1167	0.0768
7	1.062	1.0110	1.0485	0.0510	0.0135
8	1.090	0.9830	1.0220	0.1070	0.0680
9	1.056	1.0194	1.0568	0.0366	0.0008
10	1.051	0.9674	1.0071	0.0836	0.0439
11	1.057	0.9698	1.0091	0.0872	0.0479
12	1.055	0.9872	1.0260	0.0678	0.0290
13	1.050	0.9936	1.0323	0.0564	0.0177
14	1.036	0.9863	1.0254	0.0497	0.0106
MAE				0.0633	0.0280
Max.				0.1167	0.0768

TABLE-6: Bus Angles Estimates for **case-3**

Bus No.	Angle (Act.)	Angle (Con.1)	Angle (Con.2)	Abs Error (Con.1)	Abs Error (Con.2)
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	-0.0871	-0.0988	-0.0912	0.0117	0.0041
3	-0.2225	-0.2496	-0.2310	0.0271	0.0085
4	-0.1788	-0.1946	-0.1809	0.0158	0.0021
5	-0.1529	-0.1659	-0.1543	0.0130	0.0014
6	-0.2521	-0.2682	-0.2491	0.0161	0.0030
7	-0.2310	-0.2531	-0.2342	0.0221	0.0032
8	-0.2310	-0.2571	-0.2381	0.0261	0.0071
9	-0.2587	-0.2707	-0.2522	0.0120	0.0065
10	-0.2624	-0.2741	-0.2550	0.0117	0.0074
11	-0.2593	-0.2731	-0.2536	0.0138	0.0057
12	-0.2670	-0.2845	-0.2638	0.0175	0.0032
13	-0.2676	-0.2843	-0.2637	0.0167	0.0039
14	-0.2805	-0.2923	-0.2715	0.0118	0.0090
MAE				0.0154	0.0047
Max.				0.0271	0.0090

It is observed that the proposed estimator outperformed the WLS based Normal Equations estimator both an account of maximum error and MAE for voltage magnitude and bus angles.

TABLE-7 talks about the computational time for both the approaches namely WLS based Normal Equation method and WLS based Singular Value Decomposition method.

TABLE-7: Computational time for both WLS methods

TEST CASE	WLS based Normal Equation method	WLS based SVD method
Case (1)	0.057085 Seconds	0.0284168 Seconds
Case (2)	0.154832 Seconds	0.135735 Seconds
Case (3)	0.027642 Seconds	0.018413 Seconds

V. Conclusion

In this paper the development of a new algorithm for static state estimation based on the SVD method has been presented and tested for IEEE 14-bus system. The performance of the proposed Singular Value Decomposition based estimator is compared with WLS based conventional method on basis of time and accuracy. In the first case, the SVD estimator provides better results even communication noise is introduced in the measurement set. In the second case, when both the estimators run for Gross Error presented in the set of measurement data, the SVD approach gives better solution than the Normal Equations. Meanwhile, in the third case for the topological error in the measurement data; the SVD based technique proves its robustness by giving the good results close to the desired one. From the **TABLE-7**, it is cleared that the computational time for SVD algorithm is comparatively less rather than the Normal Equation algorithm for all of the above tested cases.

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