



## COLLAPSE LOAD CALCULATION FOR AXISYMMETRICALLY LOADED CIRCULAR PLATES OF UNIFORM THICKNESS

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**Abstract** — The general problem of limit analysis is the determination of the flow limit for a given structure under given types of loading ; the load intensity at the flow limit is called the “ load carrying capacity ” of the structure. This research is concerned with the determination of the load carrying capacity of thin circular plates made of a plastic-rigid material that obeys TRESCA’S yield condition and the associated flow rule. The discussion is restricted to rotationally symmetric types of loading and edge support : both simply supported and built-in plates are considered, and the load is supposed to be uniformly distributed over either a central circular region or an annular region that extends up to the edge. For conciseness, the detailed analysis is presented only in certain typical cases, and otherwise only the final results are given. The problems studied in this paper are, of course, only particular cases of the general problem of determining the load carrying capacity of a plate of arbitrary shape under an arbitrary type of loading. However, it is felt that from the study of such particular cases experience can be gained that will suggest ways of attacking the general problem.

**Keywords-** Classical method, Circular plate, Axisymmetrically loaded plate, Tresca’s yield theory.

### I. INTRODUCTION

The concept of limit analysis was introduced by structural engineers in connection with problems concerning beams and frames (see, for instance, KIST 1920 ; VAN DEN BROEK 1948 ; BAKER 1949). Only recently has the general theory of limit analysis been developed (HILL 1951 ; DRUCKER, PRAGER and GI-EENBERG 1952 ; PRAGER 1952) as a chapter of the mathematical theory of perfectly plastic solids. This development has led not only to new methods for the limit analysis of beams and frames (see, for instance, SYMONDS and NEAL 1951) but also to the application of limit analysis to other fields (HEYMAN 1951 ; WEISS, PRAGER and HODGE 1952 ; ONAT and PRAGER 1953 ; PELL and PRAGER 1951). As HILL (1951) has pointed out, the principles of limit analysis are most conveniently formulated for a perfectly plastic material that is rigid whenever the stress is below the yield limit.

### CLASICAL SOLUTIONS FOR SIMPLY SUPPORTED PLATES

Case: 1 Uniformly Distributed load

Step-1. Type of loading: Plate loaded with uniformly Distributed load on central Diameter 2C.

Step-2 Figure.

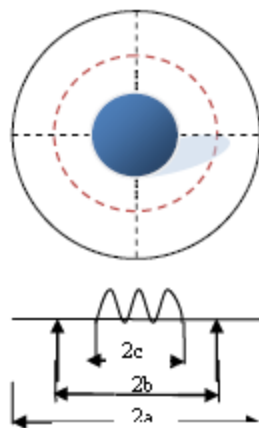


Fig. 1 Central Patch load of UDL upto Diameter 2C on overhanging plate

**Step 3.** Equation of equilibrium used

Section is taken at a distance of  $r$  from centre of plate within  $2b$  i.e.  $0 < r < b$

$$\frac{d}{dr}(r * M_r) - M_\theta = -Q_r * r$$

**Step 4.** Calculation of  $Q_r$  for  $r < b$

$$\frac{d}{dr}(r * M_r) - M_\theta = -\frac{1}{2}qr^2$$

Integration w.r.t  $dr$

$$(r * M_r) - r * M_\theta = -\frac{1}{6}qr^2 + C_1$$

**Step 5.** boundary condition used to get constant of integration

$$\text{At } r=0, M_r=M_\theta=M_y$$

& therefore  $C_1=0$

$$(r * M_r) - r * M_y = -\frac{1}{6}qr^2$$

$$\text{At } r=c, M_r=M_c$$

$$M_c = -\frac{1}{6}qc^2 + M_y$$

Section 2.2,  $c < r < b$

$$c < r < b, M_r=M_\theta$$

$$\frac{d}{dr}(r * M_r) - M_\theta = -\frac{1}{2}qc^2$$

$$(r * M_r) - r * M_\theta = -\frac{1}{2}q * r * c^2 + C_2$$

**Step 6.** boundary condition used to get constant of integration,

$$\text{At } r=c, M_r=M_c$$

$$C * \left[ -\frac{1qc^2}{6} \right] + \frac{1}{2}qc^2 = C_2$$

$$\frac{1}{3}qc^2 = C_2$$

$$(r * M_r) - r * M_\theta = -\frac{1}{2}qc^2 + \frac{1}{3}qc^2$$

$$\text{At } r=b, M_r=M_b$$

$$M_b = -\frac{1}{2}qc^2 + \frac{1}{3}qc^2$$

Section 3-3  $b < r < a$ ,  $Q_r = 0$

$$\frac{d}{dr}(r * M_r) - M_\theta = -Q_r * r$$

Integrating

$$(r * M_r) - r * M_\theta = 0 + C_2$$

At  $r=b$ ,  $M_r = M_b$

$$b(M_b - M_y) = C_2$$

$$b * \left[ -\frac{1qc^2}{2} \right] + \frac{1}{3b} qc^3 = C_2$$

$$C_2 = -\frac{1}{2} qbc^2 + \frac{1}{3} qc^3$$

$$(r * M_r) - r * M_y = -\frac{1}{2} qbc^2 + \frac{1}{3} qc^3$$

Collapses at  $r=a$ ,  $M_r = 0$

$$-a M_y = qc^2 \left( \frac{c}{3} - \frac{b}{2} \right)$$

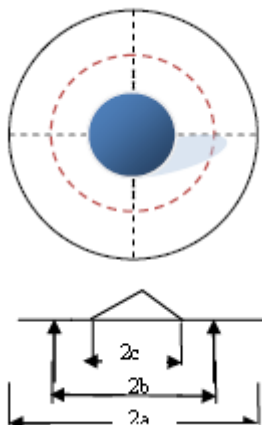
$$-a M_y = qc^2 \left[ \frac{2c-3b}{6} \right]$$

$$q = -\frac{6aM_y}{c^2(2c-3b)} = \alpha * \frac{M_y}{a^2}$$

Table: 1 multiplying Coefficient of  $M_y/a^2$  for  $\frac{b}{a} = 0.6$

$\frac{r}{a}$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.0	3370.787	8522.73	3831.42	2180.23	1411.74	992.04	757.44	571.45	457.25	375.00
0.1	313.84	247.09	230.54	201.40	177.78	158.34	142.20	128.40	117.05	107.14
0.2	98.59	91.15	84.44	78.91	73.85	69.34	65.32	61.72	58.48	55.54
0.3	52.91	50.31	48.33	44.34	44.53	42.87	41.35	39.95	38.47	37.50
0.4	34.42	35.43	34.52	33.49	32.92	32.22	31.58	31.00	30.48	30.00
0.5	29.57	29.20	28.84	28.58	28.34	28.14	27.98	27.87	27.80	27.78
0.6	924.73	-27.47	-27.04	-25.20	-24.14	-23.50	-22.19	-21.28	-20.42	-19.59
0.7	21994.04	-18.04	-17.81	-14.45	-14.00	-15.58	-14.77	-14.20	-13.45	-13.13
0.8	402255.82	-12.14	-11.99	-11.22	-10.80	-10.51	-9.99	-9.41	-9.24	-8.89
0.9	5829081.81	-8.22	-8.13	-7.41	-7.31	-7.11	-6.74	-6.50	-6.24	-6.00

Case: 2 Triangular Load



$$q = \frac{12}{k_2 2(2k_1 - k_2)} * \frac{M_y}{a^2}$$

Fig. 2 Central Patch load of Triangular load upto Diameter 2C on Overhanging Circular plate

#### REFERENCES

- [1] BEKIR TEKINALP, "Elastic-Plastic Bending Of A Built-In Circular Plate Under A Uniformly Distributed Load", *Journal of the Mechanics of Solids*, Vol.5, pp. 135 to 142
- [2] H. G. HOPKINS and A. J. WANG, "Load-carrying capacities for circular plates of perfectly-plastic material with arbitrary yield condition", *Journal of the Mechanics and Physics of Solids*, 1954, Vol. 8, pp. 117 to 129.
- [3] H. G. HOPKINS and W. PRAGER, "The load carrying capacities of circular plates", *Journal of the Mechanics and Physics of Solids*, 1953, Vol. 2, pp. 1 to 13.
- [4] K. H. LÄRME and E. T. ONAT, "A comparison of experiments and theory in the plastic bending of circular plates", *J. Mech. Phys. Solids*, 1962. Vol. 10, pp. 801 to 811
- [5] W. FLUGGE and J. C. GERDEEN, "Collapse Of A Simply Supported Circular Plate Under Two Uniform Loads", *Int. J. Solids Structures*, 1967. Vol. 3, pp. 671 to 689.
- [6] Text book, "Theory of plates and Shells" by S. Timoshenko.