



## An Assessment of Bed Load Equation Based On Alluvial River Data

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**Abstract -** There are many equations to determine the bed load transport rate. In this study, applicability of the sediment transport equations of Meyer Peter and Muller (1948), Brown (1950), Bagnold (1966), Hassanzadeh (2007) had been tested. This was done using a data set consisting of total 81 bed load transport values from two river reaches such as Chulitna River and Clearwater River. Results shows that only Brown (1950) equation delivered the best estimate of bed load of Chulitna River. The Bagnold (1966) and Hassanzadeh (2007) equations estimated the bed load more than the observed rate and Meyer Peter and Muller (1948) estimated the transport rate of bed load less than the observed rate for Chulitna River. Meyer Peter and Muller (1948) gave better agreement with measured data of Clearwater River. The high scattering in computed results by Brown (1950), Bagnold (1966) and Hassanzadeh (2007) formulas, cleared that these equations were not appropriate for estimate of bed load of Clearwater River.

**Keywords -** Sediment, bed load transport, transport equation, transport rate, shear stress

### I. INTRODUCTION

It is important to study the sediment transport mechanisms and transport capacity of streams in river hydraulics and geomorphology due to constant erosion process and sediment transport processes in natural rivers. Sediment transport process and sedimentation in river create many hydraulic problems including changing the cross-section of the river, affected the life of hydraulic structure and their reservoir capacity. It is important to study the various methods to predict the bed load transport in river engineering projects. Bed load transport prediction is vital for river engineering projects such as dam development, fluvial geomorphology, eco-hydrology, environmental surveys and management and hazard predictions [12].

Bed load transport can be described as the physical process in alluvial stream for construction and maintenance of geometry of dynamically stable channel which demonstrate not only quantity of water but also sediment delivered from the watershed [15]. Bed load transport is a fundamental connections of river hydraulics against river forms. Bed load transport occurs by interaction of turbulent flow structure with material of bed surface [4, 5]. Bed load is that kind of load which is moved by rolling, sliding and bouncing along with stream bed. There are two modes of bed load transport in which one mode of movement is by rolling and sliding with the stream bed is called contact load and second mode of movement is caused by hopping along the bed is called saltation load [6]. These modes of movement occur simultaneously in bed load transport and it is difficult to separate them completely.

Sediment transport in natural streams has been broadly considered in past and there are many theoretical and empirical equations that can be utilized with reasonable accuracy to predict the transport rate. Generally two approaches was considered in the concept of bed load transport. The first approach is based on critical variables such as velocity, shear stress, and stream power. The second approach is about probabilistic approach which gives new understanding into the bed load transport process. A number of equations have been developed to describe the bed load. The examples of such equations include Du boys (1879), Shields (1936), Meyer Peter and Muller (1948), Bagnold (1966) etc. The researchers have tested the bed load formula based on laboratory data and field data. Mahmood (1980) has tested twelve bed load equation in his study using 97 field measurements of sediment and hydraulic variables in Missouri River [9]. Bathurst et al (1987) compared performance of the six bed load equation for steep mountains rivers and found that Schoklitsch equation gave better results [2]. Gomez and Church (1989) assessed twelve bed load equation for gravel bed river using 410 data sets of flume and field measurements [7]. They found that due to complexity of transport phenomena, none of the equations gives better results. Reid et al (1996) considered the several formula on Negev Desert, Israel and found that Meyer Peter and Muller (1948) and Parker (1990) yields best only on gravel bed river [13, 14]. The objective of the paper is to study the applicability and performance of the commonly used formula in calculating the bed load transport rate in two rivers.

## II. DATA USED IN THIS STUDY

In this study, the field data published by the Geological Survey of the United States of America has been used to assess the performance of the selected formulas. These data are taken from two River stations in the United States including Chulitna River below Canyon near Talkeetna, Alaska (1982-1985) and Clearwater River at Spalding Idaho (1972 to 1979) [16]. Data regarding measured bed load transport rate, flow discharge, flow depth, flow velocity, channel slope, and sediment diameter used in the formula. Table 1 and Table 2 presents the summary of hydraulic data on Chulitna River near Talkeetna, Alaska and Clearwater River at Spalding Idaho respectively.

**Table 1. Summary of hydraulic data of Chulitna River near Talkeetna, Alaska**

Bed load transport rate (kg/s)	D50 (m)	D90 (m)	Flow discharge(m <sup>3</sup> /s)	Flow velocity (m/s)	Flow depth (m)	Slope	Water surface width (m)
21.9 - 130	0.012	0.062	212 - 1350	1.2 - 2.5	1.7 – 3.1	0.00039 – 0.001	98.5 - 136

**Table 2. Summary of hydraulic data of Clearwater River at Spalding Idaho**

Bed load transport rate (kg/s)	D50 (m)	D90 (m)	Flow discharge(m <sup>3</sup> /s)	Flow velocity (m/s)	Flow depth (m)	Slope	Water surface width (m)
8.66 – 0.0109	0.02	0.14	312 - 2740	0.70 – 3.0	3.5 – 5.1	0.000098 – 0.000359	125 - 145

## III. BED LOAD TRANSPORT EQUATIONS

The following bed load equations were selected for calculating the bed load transport rate are presented in table 3.

**Table 3. List of bed load transport equation used in this study**

Author	Bed load formula
Meyer Peter and Muller (1948)	Energy slope approach
Hassanzadeh (2007)	Regression approach
Brown (1950)	Probability approach
Bagnold (1966)	Stream power approach

### Meyer Peter and Muller equation (1948)

Meyer Peter and Muller proposed an empirical equation for predicting bed load transport which is fully analytical and dimensionally homogeneous [11]. Meyer Peter and Muller found that the total shear stress is not responsible for sediment transport in case of an undulated bed, a part of shear stress is used up in overcoming the form resistance and the bed load transport is a function of remaining part. Meyer Peter and Muller formula expressed as:

$$q_b = \emptyset * \rho_s * (g * \Delta * d^3)^{0.5}$$

Where  $\emptyset$  is dimensionless sediment transport parameter and is expressed as:

$$\emptyset = 8 * (\tau'_s - 0.047)^{1.5}$$

Where  $\tau'_s$  is a dimensionless shear stress,  $\rho_s$  is sediment density and  $d$  is sediment diameter and  $\Delta = \frac{\rho_s - \rho}{\rho}$ .

### Brown equation (1950)

Brown has modified the Einstein approach for bed load prediction which is based on probabilistic approach [2]. The set of equation used in this study are as follows:

$$q_{sv} = q_* * F_1 * \sqrt{\left(\frac{y_s}{y} - 1\right) * g * D_{50}^3}$$

$$F_1 = \left[ \frac{2}{3} + \frac{36 * \vartheta^2}{\left(\frac{\gamma_s}{\gamma} - 1\right) * g * D_{50}^3} \right]^{0.5} - \left[ \frac{36 * \vartheta^2}{\left(\frac{\gamma_s}{\gamma} - 1\right) * g * D_{50}^3} \right]^{0.5}$$

$$q_* = 2.15 \exp\left(\frac{-0.391}{\tau_*}\right) \text{ When } \tau_* < 0.09$$

$$q_* = 40 * \tau_*^3 \text{ When } \tau_* > 0.09$$

$$\tau_* = \frac{\gamma * R * S}{(\gamma_s - \gamma) * D_{50}}$$

$q_{sv}$  is the bed load discharge in volume per unit width,  $q_*$  is the dimensionless volumetric bed load transport rate per unit width,  $F_1$  is parameter of fall velocity,  $D_{50}$  is median sediment diameter.

### Bagnold equation (1966)

Based on the concept of energy balance in alluvial flow, Bagnold has proposed new dimensionless equations for estimating the bed and suspended load transport rate [1]. For the calibration of proposed model and derivation of functional relationship for bed and suspended load transport efficiency coefficients, Bagnold used the laboratory flume and field data. From the physical point of view the stream power supplies energy for the fluid flow, which is used partly in transportation of bed load and the suspended load. Fluid flow, on the other hand perform work on the sediment particles to keep them in movement this work is proportional to the stream power. According to Bagnold, bed load transport rate in dry weight per unit width of flow can be written as:

$$q_b = \frac{\rho_s}{\rho_s - \rho} * \tau_o * V * (e_b / \tan \alpha)$$

$\tau_o$  is bed shear stress,  $V$  is flow velocity,  $e_b$  is bed load transport rate efficiency,  $\tan \alpha$  is coefficient of inter angular friction

### Hassanzadeh equation (2007)

More recently, based on the dimensional analysis and the Buckingham  $\Pi$  – theorem, Hassanzadeh (2007) has presented a dimensionless semi-empirical equation on the bed load [8]. This latest dimensionless bed load equation has been given as a function of the hydrodynamic-immersed gravity force ratio.

$$q_b = 24 * f^{2.5} * (g * \Delta * d^3)^{0.5}$$

Where  $f = \frac{(\rho * h * S)}{(\rho_s - \rho) * d}$

Where  $\rho$  = fluid density,  $\rho_s$  = sediment density,  $h$  = flow depth,  $S$  = channel slope and  $\Delta = \frac{\rho_s - \rho}{\rho}$ .

## IV. STATISTICAL ANALYSIS

In order to determine the accuracy of selected formulas, predicted values are compared with measured values using two Statistical methods below.

### Discrepancy ratio

It is the measure of an equation to replicate data accurately. It is the ratio of a predicted to the observed bed load discharge. If this ratio is one, the equation exactly predicts the measured rate. If the ratio is less than one or greater than one the equation under or over predicts measured data respectively. Mathematically,  $r = q$  (predicted)/ $q$  (observed)

### Standard deviation

It is a measure that is used to quantify the amount of variation of a set of data values. A low standard deviation indicates that the data points tend to be close to the mean, while a high standard deviation indicates that the data points are spread out over a wider range of values.

$$\sigma = \sqrt{\sum_{i=1}^n (Ri - \bar{R})^2 / (n - 1)}$$

Where  $\bar{R}$  = Averaged variation coefficient

$$\bar{R} = \sum_{i=1}^n Ri / n$$

## V. RESULTS

The bed load transport rate was calculated by using different bed load transport equation which is listed in table 3. Bagnold (1966) and Hassanzadeh (2007) equation predicted bed load transport rate of Chulitna River more than

measured values as shown in figure 1 (c) and (d). The prediction of bed load transport rate by Meyer Peter and Muller (1948) under predicts the value for Chulitna River as show in figure 1 (a). From the figure 1 (b) it is observed that most of data lies near the line of equality hence Brown (1950) gave better results among the selected equation for Chulitna River. From the figure 2 (b) and (c) it is observed that Bagnold and Brown equation under predicts the bed load transport rate for Clearwater River. The predicted bed load transport rate by Hassanzadeh (2007) under predicts as well as over predicts the measured values for Clearwater River as shown figure 2 (d). Only Meyer Peter and Muller (1948) equation delivers better results for Clearwater River compared to other selected equation as shown in figure 2 (a). Results of correlation of calculated and measured bed load transport rate of Chulitna River and clear water River are outlined in table 4 and table 5 respectively. Since bed load is computed by empirical equations for averaged River depth and width while morphological condition of a natural River are varying with time, thus a difference is always expected between the computed bed load and observed bed load.

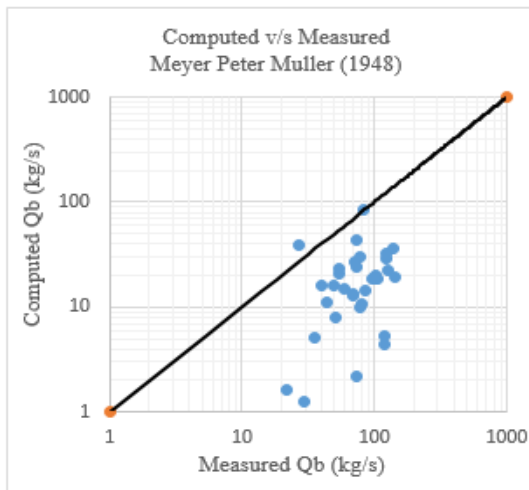


Fig 1 (a)

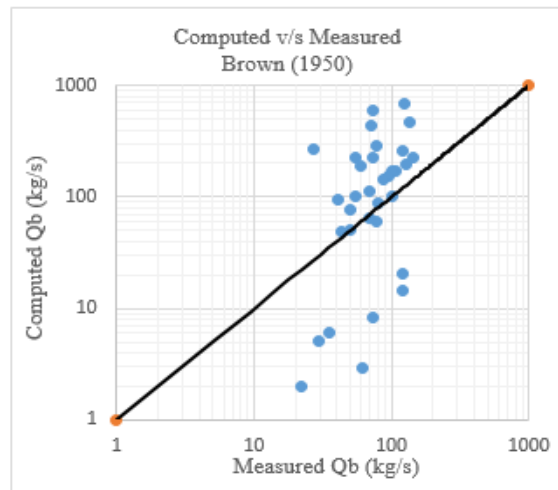


Fig 1 (b)

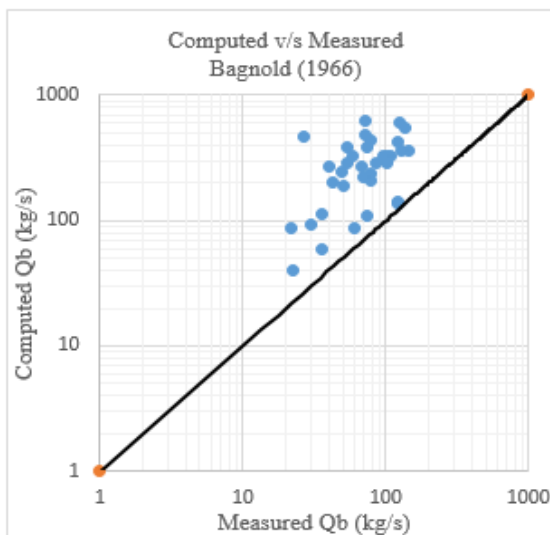


Fig 1 (c)

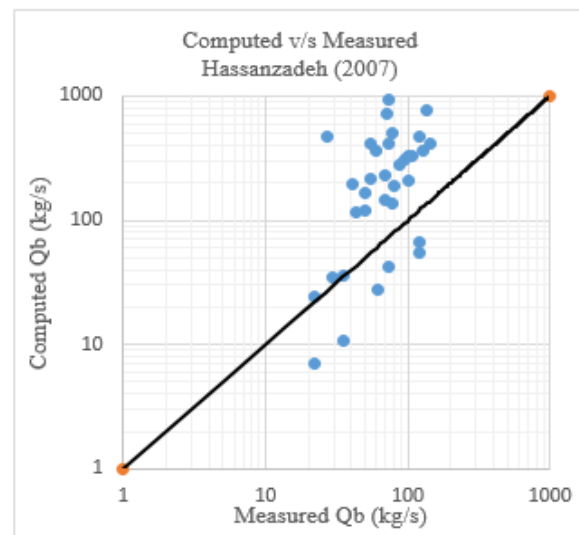


Fig 1 (d)

**Fig. 1 Comparison between computed and measured bed load transport rate of Chulitna River data: (a) Meyer Peter and Muller (1948) (b) Brown (1950) (c) Bagnold (1966) (d) Hassanzadeh (2007)**

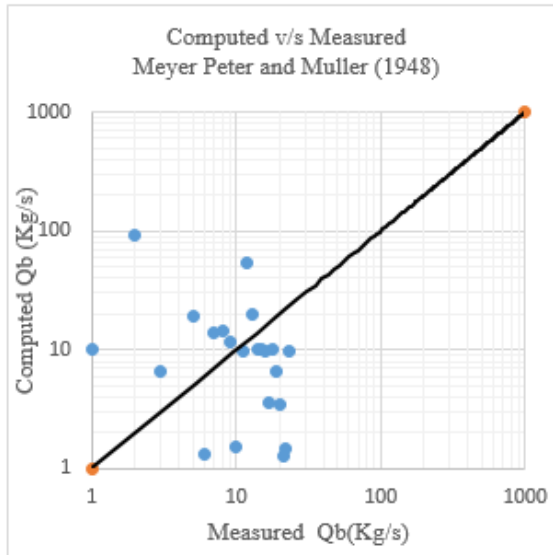


Fig 2 (a)

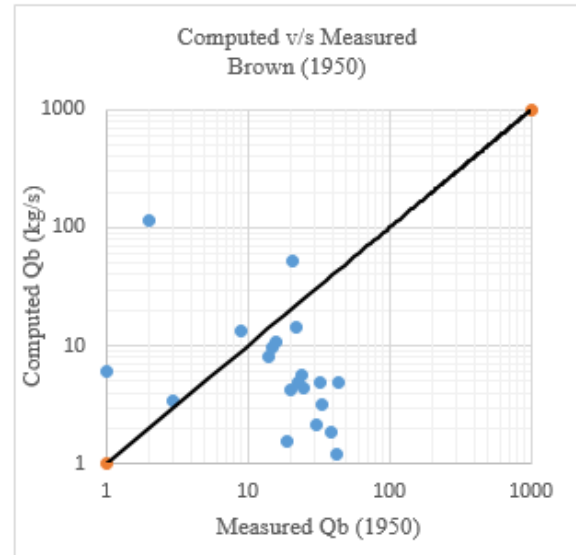


Fig 2 (b)

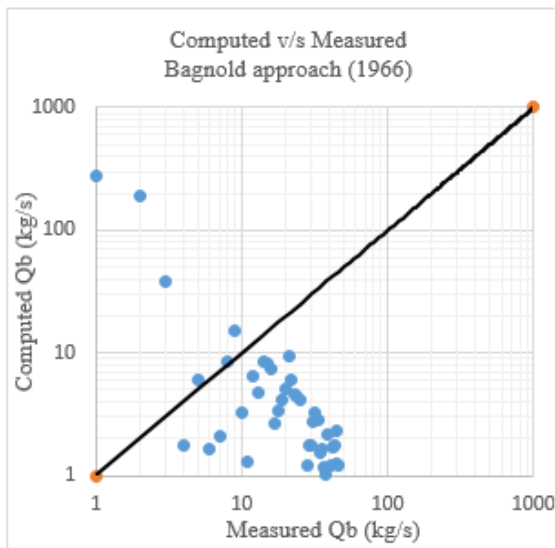


Fig 2 (c)

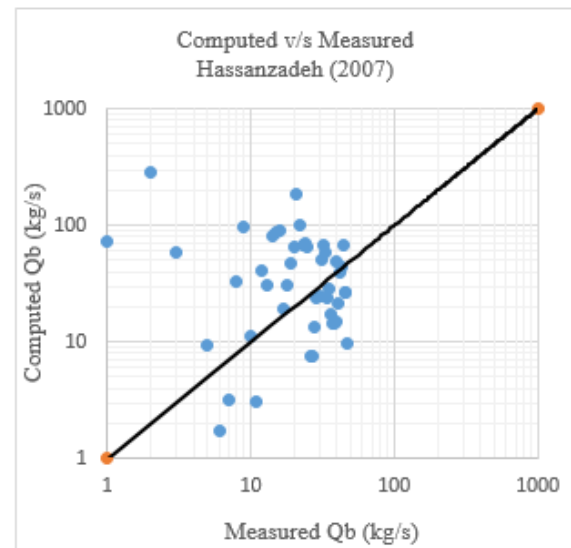


Fig 2 (d)

Fig. 2 Comparison between computed and measured bed load transport rate of Clearwater River data: (a) Meyer Peter and Muller (1948) (b) Brown (1950) (c) Bagnold (1966) (d) Hassanzadeh (2007)

Table 4. Variation coefficient for selected equations for Chulitna River

Sr. No.	Bed load transport equation	Average of variation coefficient	Standard deviation
1	Meyer Peter and Muller (1948)	0.26	0.27
2	Brown (1950)	4.32	3.22
3	Bagnold (1966)	4.73	3.83
4	Hassanzadeh (2007)	6.31	4.46

Table 5. Variation coefficient for selected equations for Clearwater River

Sr. No.	Bed load transport equation	Average of variation coefficient	Standard deviation
1	Meyer Peter and Muller (1948)	3.94	3.48
2	Brown (1950)	1.48	2.99
3	Bagnold (1966)	8.69	27.64
4	Hassanzadeh (2007)	24.12	25.20

## VI. RESULT ANALYSIS

Various empirical equations including Meyer Peter and Muller (1948), Brown (1950), Bagnold (1966), Hassanzadeh (2007) used to determine the bed load transport rate using field data of Chulitna River and Clearwater River.

- From the selected formulas, only Brown (1950) equation is found to predict satisfactorily the bed load of Chulitna River.
- Bagnold (1966) and Hassanzadeh (2007) equations over predicted the bed load while Meyer Peter and Muller (1948) under predicted the bed load of Chulitna River.
- For Brown (1950) equation, discrepancy ratio is close to one, value of average variation coefficient is 4.32 and value of standard deviation of variation coefficient is 3.22.
- The best estimate of bed load of Clearwater River were produced by Meyer Peter and Muller (1948) equation.
- For Meyer Peter and Muller (1948) equation, discrepancy ratio is close to one, value of average variation coefficient is 3.94 and value of standard deviation of variation coefficient is 3.48.
- Bagnold (1966) and Brown (1950) equation under predicted the bed load of Clearwater River while Hassanzadeh (2007) equation under predicted as well as over predicted the bed load of Clearwater River.
- Greater precision of a formula does not ensure that the formula is better than others under all flow and sediment conditions. The accuracy of a formula may change depending on channel bed slope, particle size diameter, and other hydraulic and sedimentological data.

## VII. CONCLUSION

In this study, four bed load equations are evaluated using measured bed load data of Chulitna River and Clearwater River. Among the selected equations only Brown (1950) yields good results for Chulitna River data. The Meyer Peter and Muller (1948) equation performs well with measured data of Clearwater River data. The results given by different equations were different and were significantly different than the measured values which can be correlated to the wash load from loose formations of the basin not considered in empirical equations.

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