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Flow Of A Viscous Fluid Through A Porous Circular And It's Surrounding Porous Medium

Toran Kumar[1] and B.N. Upadhyay[1]

P.G. Department of Mathematics

Gaya College, Gaya - 823001

Magadh University (Bodh-Gaya) Bihar.

Abstract

In the present paper, we have studied the viscous flow through a circular pipe which is bounded by a porous medium fully saturated with viscous fluid in the presence of an applied magnetic field. Two cases have been taken into account. (i) When porous medium is unbounded (ii) When it is bounded by another Co-axial impervious circular cylinder. It is assumed that the flow inside the pipe in which a clear fluid is there, is governed by Navier-stokes equation and in the porous medium outside the cylinder the flow is given by Brinkmen [4], At the interface of two types of flows, the matching conditions suggested by Ochoa-Tapia and Whitaker [7] have been followed. The governing equations have been solved and the solution for velocity in both cases with porous media have been deduced. The results for different values of magnetic field in both cases have been observed and discussed graphically.

Keywords: *Viscous fluid, porous medium, magnetic field, Magneto hydrodynamic flow.*

1. INTRODUCTION:

The modern technology has stimulated the interest of several researchers in the field of flow studies which involves the interaction of several phenomenon. A type of such investigation is presented when a viscous fluid flow over a porous surface, because of its applications in many enginerring problems. Such a flow of liquid in a porous bearing Joseph and Tao [1], in the field of water in river beds, in petroleum technology to explore the movement of natural gas, oil and water through oil reservoirs, in chemical engineering for filtration and purification processes. A large number of geophysical applications of fluid flow in porous media have been studied by Cunningham and Williams [2]. In these problems, the fluid flows through two regions, one region there is a clear fluid and in the other region medium there is a porous solid. The two flows are matched at the interface by appropriate conditions.

The mathematical theory of a fluid flow through a porous medium was taken into consideration by Darey [3], later on Brinkman [4] suggested some modification of Darey's law for porous medium. Numerous workers in this field have studied problems of flow of fluid in two regions. It is assumed that the flow inside circular pipe, in which a clear fluid is there, is governed by Navier – stokes equation and in porous medium outside the cylinder the flow is given By Brinkman equation. Srivastava et. al [5] have studied the flow in the porous medium induced by torsional oscillation of a disk in a clear fluid near its surface by using the above analysis.

The study of magnets hydrodynamic flows through porous medium has been studied by various researchers [6-7]. The influence of blood flows has been studied by Suri and Suri [8], Sud et. al [9] and Yamamato [10]. Sud and Sekhan [11] have improved a model of blood flow through a human arterial system subjected to a steady magnetic field. Srivastava et. al. [12] have developed a model for the flow of viscous fluid through a circular pipe and its surrounding porous medium bounded by a rigid cylinder and unbounded regions.

The present paper deals with the study of viscous fluid flow through a circular cylindrical pipe surrounded by a porous medium in the presence of applied magnetic field. Two cases have been considered.

(i) The porous medium is unbounded.

(ii) When it is bounded by another co-axial imprevious circular cylinder. It is assumed that the flow inside the pipe, in which the clear fluid is there, governed by Navier-stokes equation and in porous medium outside the cylinder the flow is given by Brinkman equation [4]. Ochoa-Tapia and Whitaker [7] suggested the matching condition at the interface of two flows.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

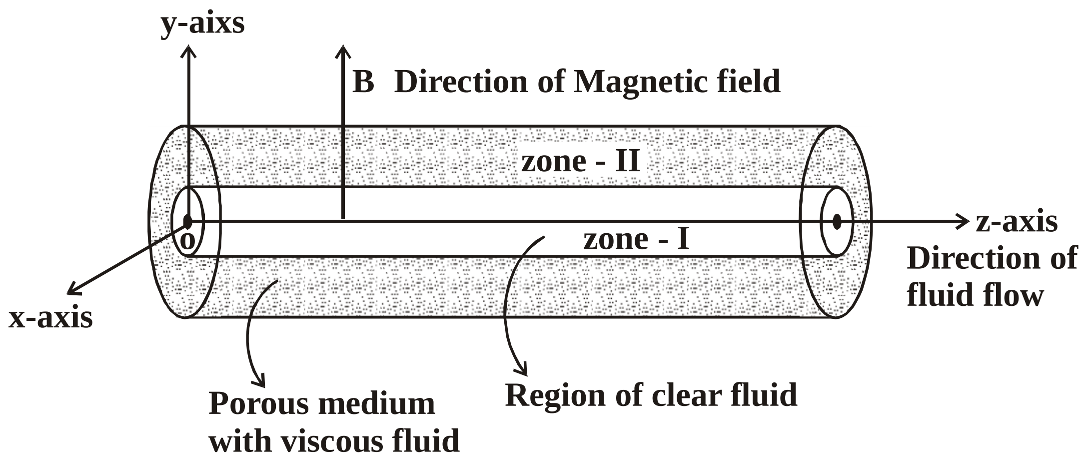

Fig. 1

Consider the flow of incompressible viscous fluid through a circular pipe of radius r' = a and through its surrounding porous medium of infinite extent $(r' \geq a)$ under uniform pressure gradient in the direction of the axis of pipe. The porous medium is founded by another impervious co-axial circular cylinder of radius $r' = \lambda a (\lambda > 1)$. We also consider a magnetic field in the direction normal to the common axis of cylinders. The region inside the circular pipe in which clear fluid flows is represented by zone-I and region outside the cylindrical pipe $(r' \geq a)$ which is occupied by the porous medium with viscous fluid is designated by zone-II. We select a cylindrical polar system of referance frame with co-ordinates (r', θ, z') where z' axis coincides with the common axis of cylindrical pipes. We assume that in both zones the velocity components of fluid in the direction of r and □ is zero. The velocity component in direction of z' is denoted by $u'(r')$ and $v'(r')$ in zone-I and zone-II respectively. Both u' and v' are functions of r' only. We further assume that flows in zone-I and zone-II are governed by Navier and Stokes together with Brinkman [4] equations. With these assumptions we find that the pressure in both the zones are function of z' alone and are the same.

The equation of motion in z'-direction in zone – I is given by

$$\frac{dp}{dz'} = \mu\left[\frac{d^2u'}{dr'^2} + \frac{1}{r'}\frac{du'}{dr'}\right] + \vec{J}' \times \vec{B}', \quad 0 \leq r' \leq a \quad ......(2.1)$$

The equation of motion in zone-II in z' direction is given by

$$\frac{dp}{dz'} = \mu_e \left[\frac{d^2 v'}{dr'^2} + \frac{1}{r'} \frac{dv'}{dr'} \right] \frac{-\mu}{k} v' + \vec{J}' \times \vec{B}', \ r' > a \quad ......(2.2)$$

$$\vec{J} = \sigma(\vec{E}' + \vec{u}' \times \vec{B}') \quad .....(2.3)$$

The value of $\vec{j'} \times \vec{B'}$ for zone - I is given as

$$\vec{J}' \times \vec{B}' = -\sigma \mu_p^2 H_0^2 U_0 u' \quad .....(2.4)$$

For zone - II

$$\vec{J}' \times \vec{B}' = -\sigma \mu_p^2 H_0^2 U_0 v' \quad ......(2.5)$$

The boundary and matching conditions are given by

$$u'(r') = v'(r') \quad \textbf{at} \quad r' = a \quad ......(2.6)$$

$$\mu_e \frac{dv'}{dr'} - \mu \frac{du'}{dr'} = \frac{\mu\beta}{\sqrt{k}} v' \quad \textbf{at} \ r' = a \quad ......(2.7)$$

and u' is finite at r' = 0 ....(2.8)

v' is finite at $r' = \infty$ ....(2.9)

where □ is the viscosity of fluid in zone-I.

μ_e is the viscosity of fluid in zone-II. μ'- the velocity of fluid in z' direction in zone-I, v'- the velocity of fluid in z' direction in zone - II, k the permeability of the porous medium J' the magnetic flux, B' magnetic flux, E' electric flux, □ the electrical conductivity, □$_p$ permeability of medium, H_0 the magnetic field intensity, □ a constant which may be positive and negative.

Since □ is a constant which takes both positive and negative values and is determined experimentally for a particular porous medium (See Ochoa-Tapia and Whitaker) [7]

We write

$$\frac{dp}{dz'} = -\mu A \qquad \text{......(2.10)}$$

We take non-dimensional quantities as

$$u' = Aa^2 u, \quad v' = Aa^2 v, \quad r' = ar$$

The non-dimensional equation for zone-I is given by

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - M_0^2 u = -1 \qquad 0 \le r < 1 \qquad \text{.....(2.11)}$$

The non-dimensional equation for zone-II is given by

$$\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - Q^2 v = -\frac{1}{\gamma^2}, \qquad r > 1 \qquad \text{.....(2.12)}$$

where,

$$M_0 = \frac{\sigma\mu_p^2 H_0^2 r_0^2}{\mu}, N = \frac{\sigma\mu_p^2 H_0^2 r_0^2}{\mu_e}, \ Q^2 = \alpha^2 + N^2,$$

$$\gamma^2 = \frac{\mu_e}{\mu}, \quad \sigma = \frac{r_0}{\sqrt{k}}, \quad \alpha = \frac{\sigma}{r}$$

Non-dimensional boundary and matching conditions are written as

$$u = v \quad \text{at} \quad r = 1 \qquad \text{.......(2.13)}$$

$$\gamma^2 \frac{dv}{dr} - \frac{du}{dr} = \beta\sigma v \quad \text{at} \quad r = 1 \qquad \text{........(2.14)}$$

u is finite at $r = 0$ ....(2.15)

v is finite at $r = \infty$ ....(2.16)

3. SOLUTION OF EQUATIONS

There are two cases (i) $r \geq 1$ (ii) $\lambda \geq r \geq 1$

Case (i) : The porous medium is infinite in extent outside the pipe so the boundary conditions are (2.15) and (2.16)

The solution of (2.11) and (2.12) satisfying (2.15) and (2.16) are given by

$$u(r) = \frac{1}{M_0^2} + C.\,I_0(M_{0r}) \quad \text{in zone - I} \qquad \text{.....(3.1)}$$

$$v(r) = \frac{1}{Q^2\gamma^2} + B.\,k_0(Q_r)$$

and in zone - II ....(3.2)

where $I_0(M_{0r})$ is the is the Bessel function of first kind of order zero and $k_0(Q_r)$ is a modified Bessel function of second kind of order zero and B, C are constants to the determined by matching conditions at the interface (2.13) and (2.14). Substituting (3.1) and (3.2) in (2.13) and (2.14)

we get $$\frac{1}{M_0^2} + C.\,I_0(M_0) = \frac{1}{Q^2\gamma^2} + Bk_0(Q) \qquad \text{......(3.3)}$$

$$B\,Q\,k_1\,(Q) - C.\,M_0.I_1\,(M_0) = \beta\sigma\left[\frac{1}{Q^2\gamma^2} + Bk_0(Q_r)\right]$$

and ......(3.4)

where $k_1(Q)$ is the modified Bessel function of second kind of order one and $I_1(M_0)$ is modified Bessel function of first kind of order one. Equations (3.3) and (3.4) and solving give B and C.

$$B = \frac{(M_0^2 - Q^2\gamma^2)I_1(Q) + M_0\beta\sigma I_0(Q)}{M_0^2 Q^2\gamma^2\,[\{Q\gamma^2 k_1(Q) - \beta\sigma k_0(Q)\}\,I_0(M_0) - M_0 I_1(M_0)k_0 Q]} \quad ......(3.5)$$

$$C = \frac{(M_0^2 - Q^2\gamma^2)k_1(Q) + Q\beta\sigma k_0(Q)}{M_0^2 Q[\{Q\gamma^2 k_1(Q) - \beta\sigma k_0(Q)\}I_0(M_0) - M_0 I_1(M_0)k_0(Q)]} \quad ....(3.6)$$

Case (2) : The porous medium fills the annular region between *r = 1* and The expression for *u(r)* is zone-I is given by

$$u(r) = \frac{1}{M_0^2} + C.I_0(M_{0_r}) \quad .....(3.7)$$

and in zone-II

$$v(r) = \frac{1}{Q^2\gamma^2} + D.I_0(Q_r) + F\,k_0(Q_r) \quad ......(3.8)$$

As r = □ is radius of imparavious cylinder, *v(r)* has to satisfying the following conditions

$$v(\lambda) = 0 \quad ......(3.9)$$

The solution for second case subject to the matching and boundary conditions are obtained with the following relation for the constant involved.

$$D\,I_0(Q\lambda) + F\,k_0(Q\lambda) + \frac{1}{Q^2\gamma^2} = 0 \quad ....(3.10)$$

Substituting (3.7) and (3.8) in (2.13) and (2.14) we get the following relations

$$\frac{1}{M_0^2} + C.\,I_0(M) = D\,I_0(Q) + F\,k_0(Q) + \frac{1}{Q^2\gamma^2}$$ ....(3.11)

$$-C.M_0 I_1(M_0) + DQI_1(Q) + FQk_1(Q) = \beta\sigma\left[DI_0(Q) + Fk_0(Q) + \frac{1}{Q^2\gamma^2}\right]$$ and ...(3.12)

where $I_1(M_0)$ is modified Bessel function of first kind of order one and $k_1(Q)$ is modified Bessel function of order one and C, D, F are constant to be determined by (3.10), (3.11) and (3.12)

on solving the above equations one gets

$$D = \frac{\begin{array}{c} M_0^2 I_1(M_0)k_0(Q) - M_0 Q\gamma^2 I_0(M_0)k_1(Q) + M_0\beta\sigma I_0(M_0)k_0(Q) \\ + Q^2\gamma^2 I_1(M_0)k_0(Q\lambda) - M_0^2 I_1(M_0)k_0(Q\lambda) - M_0\beta\sigma I_0(M_0)k_0(Q\lambda) \end{array}}{\begin{array}{c} M_0 Q^2\gamma^2[\{M_0 I_1(M_0) + \beta\sigma I_0(M_0)\}\{I_0(Q)k_0(Q\lambda) - k_0(Q)I_0(Q\lambda)\} \\ + Q\gamma^2 I_0(M_0)\{k_1(Q)I_0(Q\lambda) - I_1(Q)k_0(Q\lambda)\}] \end{array}}$$

.......(3.13)

$$F = \frac{\begin{array}{c} M_0\{M_0 I_1(M_0) + \beta\sigma I_0(M_0)\}\{I_0(Q\lambda) - I_0(Q)\} \\ + Q\gamma^2\{M_0 I_0(M_0)I_1(Q) - QI_1(M_0)I_0(Q\lambda)\} \end{array}}{\begin{array}{c} M_0 Q^2\gamma^2[\{M_0 I_1(M_0) + \beta\sigma I_0(M_0)\}\{I_0(Q)k_0(Q\lambda) - k_0(Q)I_0(Q\lambda)\} \\ + Q\gamma^2 I_0(M_0)\{k_1(Q)I_0(Q\lambda) - I_1(Q)k_0(Q\lambda)\}] \end{array}}$$

.....(3.14)

$$C = \frac{\begin{array}{c} M_0^2\{I_1(Q)k_0(Q) - I_0(Q)k_1(Q)\} - Q\beta\sigma\{I_0(Q)k_0(Q\lambda) - k_0(Q)I_0(Q\lambda)\} \\ + (M_0^2 - Q^2\gamma^2)\{k_1(Q)I_1(Q\lambda) - I_1(Q)k_0(Q\lambda)\} \end{array}}{\begin{array}{c} M_0^2 Q[\{M_0 I_1(M_0) + \beta\sigma I_0(M_0)\}\{I_0(Q)k_0(Q\lambda) - k_0(Q)I_0(Q\lambda)\} \\ + Q\gamma^2 I_0(M_0)\{k_1(Q)I_0(Q\lambda) - I_1(Q)k_0(Q\lambda)\}] \end{array}}$$

....(3.15)

4. RESULTS AND DISCUSSIONS

We have observed the influence of applied magnetic field with different parameters according to the analysis of the present problem. By extending the previous works of earlier researchers we have obtained some realistic results on the topic given in the graph.

In figure (2) and figure (3) we observe that decreases the velocity of profiles in the zone - I and zone - II. In figure (4) and figure (5) we observe that the effect of magnetic field on both regions of flow is to decreases the value of M_0 but there is no change in porous medium.

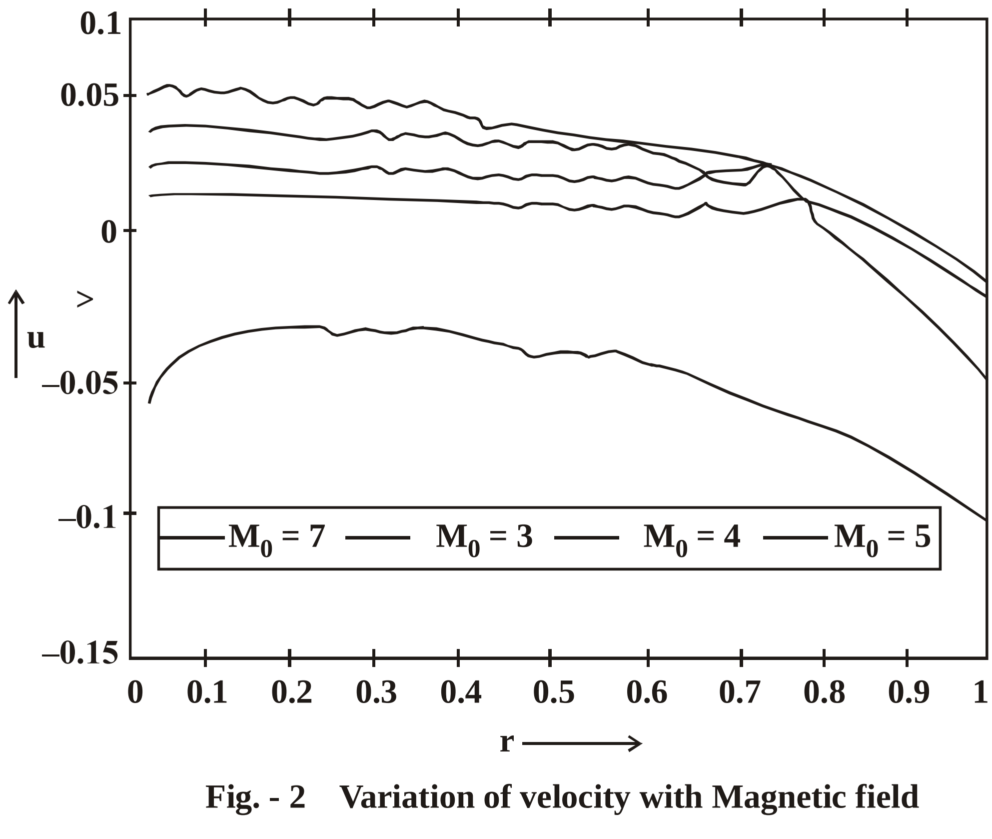

Fig. - 2 Variation of velocity with Magnetic field

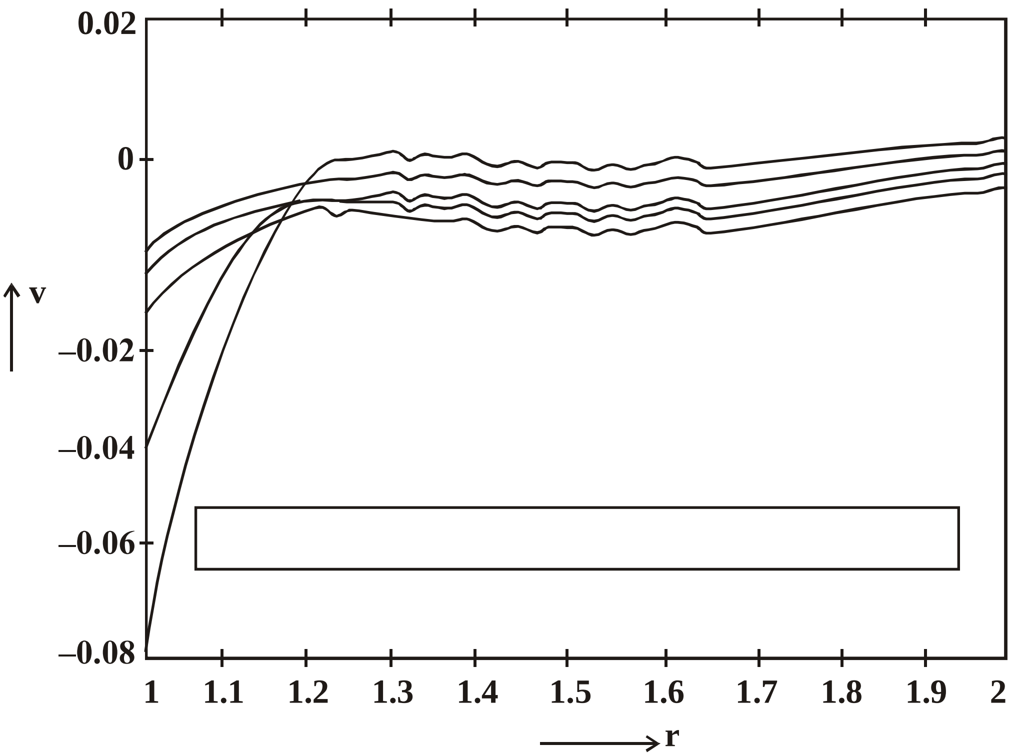

Fig. - 3 Variation of velocity with Magnetic field

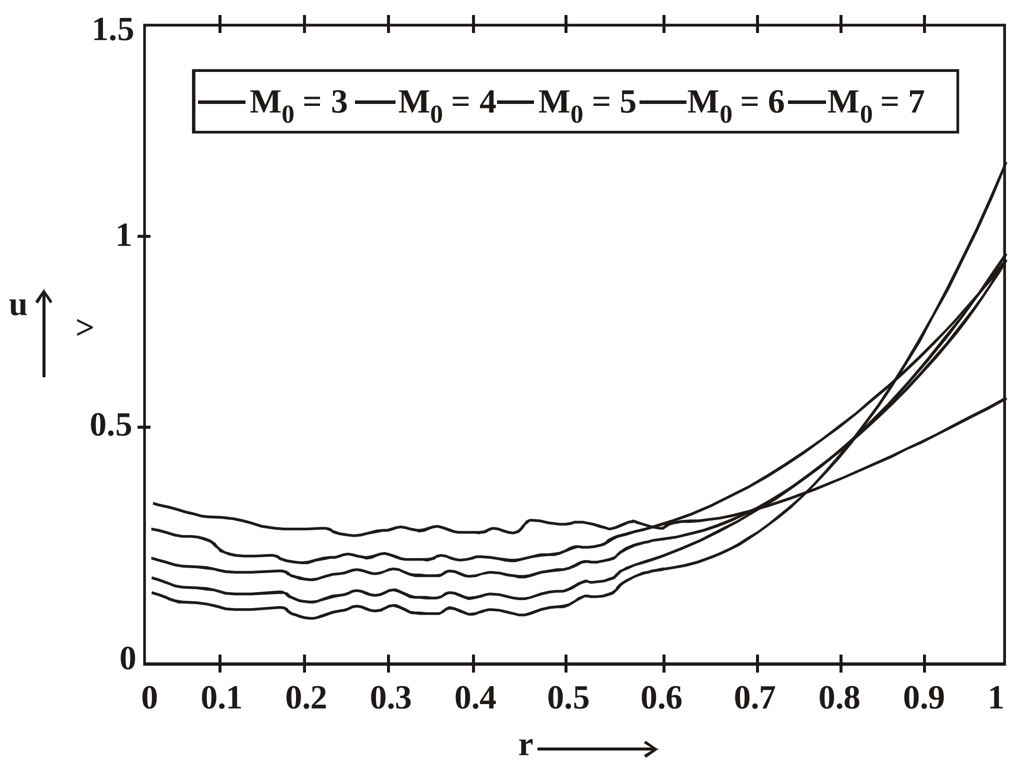

Fig. - 4 Variation of velocity with Magnetic field

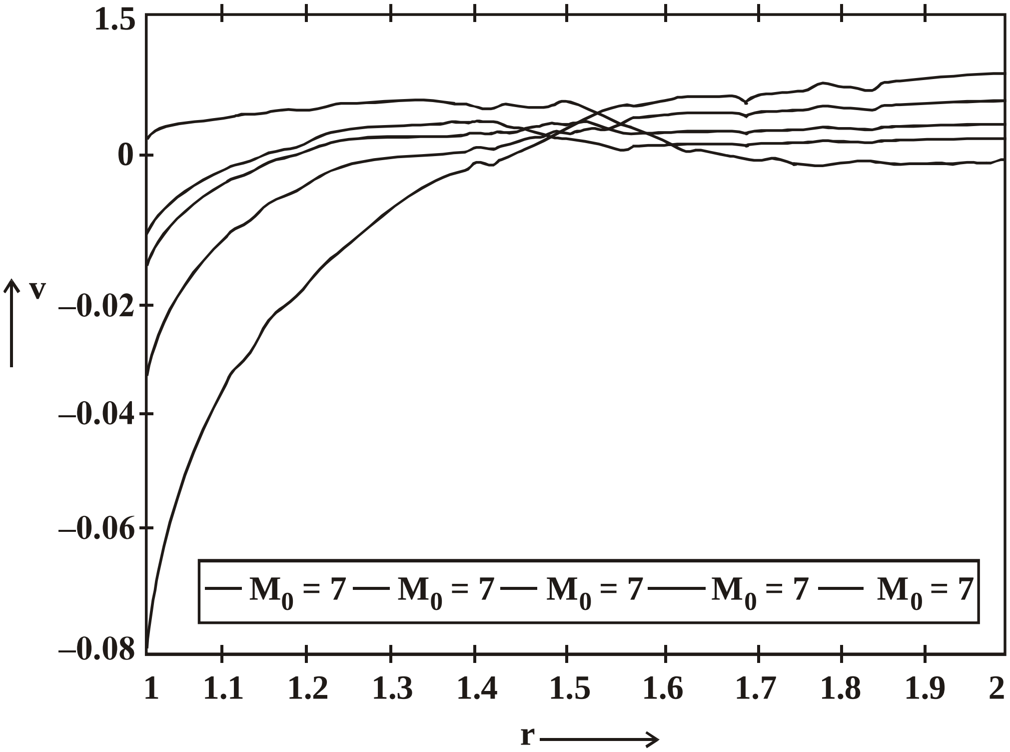

Fig. - 5 Variation of velocity with Magnetic field

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